ECS 332: In-Class Exercise # 8

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **28** / **09** / 2018			
Name	ID (last 3 digits)		
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1. For each of the following signal $g(t)$, find its (normalized) average power $P_g \equiv \left\langle |g(t)|^2 \right\rangle$.

Do not use any approximation.

Recall that (1) when $g(t) = \sum_k c_k e^{j2\pi f_k t}$ and $f_1, f_2, \ldots$ are all distinct,

$$P_g = \sum_k |c_k|^2 \quad [4.22]$$

(2) When $g(t) = \sum_k A_k \cos(2\pi f_k t + \theta_k)$ and $f_1, f_2, \ldots$ are all positive and distinct

$$P_g = \sum_k \frac{|A_k|^2}{2} \quad [4.27]$$

(a) $g(t) = 30e^{j30\pi t}$ — c_1, $f_1 = 15$

$P_g \overset{(1)}{=} |c_1|^2 = 30^2 = 900$

(b) $g(t) = 30e^{j30\pi t} + 40e^{j40\pi t}$ — c_1, $f_1 = 15$, c_2, $f_2 = 20$, $f_1 \neq f_2$

$P_g \overset{(1)}{=} 30^2 + 40^2 = 2{,}500$

(c) $g(t) = 30\cos(30t + 30^\circ)$ — A_1, $f_1 = \frac{30}{2\pi} = \frac{15}{\pi}$

$P_g \overset{(2)}{=} \frac{30^2}{2} = 450$

(d) $g(t) = 30\cos(30t + 30^\circ) + 40\cos(40t + 40^\circ)$ — A_1, $f_1 = \frac{30}{2\pi} = \frac{15}{\pi}$, A_2, $f_2 = \frac{40}{2\pi}$, $f_1 \neq f_2$

$P_g \overset{(2)}{=} \frac{A_1^2}{2} + \frac{A_2^2}{2} = \frac{30^2 + 40^2}{2} = \frac{50^2}{2} = 1{,}250$

(e) $g(t) = 30e^{j30t} + 30\cos(30t) = 30e^{j30t} + 30\left(\frac{1}{2}e^{j30t} + \frac{1}{2}e^{-j30t}\right) = 45e^{j30t} + 15e^{-j30t}$

A_1, $f_1 = \frac{30}{2\pi} = \frac{15}{\pi}$; A_2, $f_2 = \frac{-30}{2\pi} = \frac{-15}{\pi}$; $f_1 \neq f_2$

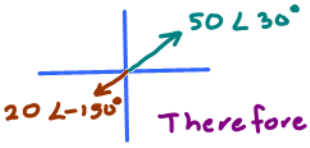
$P_g = 45^2 + 15^2 = 2{,}250$

(f) (Optional) $g(t) = 50\cos(30t + 30^\circ) + 40\cos(30t + 120^\circ) + 20\cos(30t - 150^\circ)$

Note that all three terms share the same freq. Therefore, we must combine them first. In phasor form, we have

$50\angle 30^\circ + 40\angle 120^\circ + 20\angle -150^\circ$

From the vector plot, we see that



$50\angle 30^\circ + 20\angle -150^\circ = 30\angle 30^\circ$

Therefore, we simply have to find $40\angle 120^\circ + 30\angle 30^\circ$

Note that the angle between them is 90°

So, their sum must be $\sqrt{40^2 + 30^2}\angle\theta = 50\angle\theta$ for some θ

This implies $g(t) = 50\cos(30t + \theta)$. Therefore, $P_g \overset{(2)}{=} \frac{50^2}{2} = 1{,}250$