ECS 332: In-Class Exercise # 6

Date: **12** / **09** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **"ENRPr"** = Explanation is not required for this problem.
   "ENRPa" = Explanation is not required for this part.
4. **Do not panic.**

Recall that $e^{j2\pi f_0 t} \to \boxed{H(f)} \to H(f_0)e^{j2\pi f_0 t}$

and $\cos(2\pi f_0 t) \to \boxed{H(f)} \to \frac{1}{2}H(-f_0)e^{-j2\pi f_0 t} + \frac{1}{2}H(f_0)e^{j2\pi f_0 t}$

Therefore, suppose $a\cos(2\pi f_0 t) \to \boxed{H(f)} \to b\cos(2\pi f_0 t)$.

We know that $b = aH(-f_0) = aH(f_0)$

$\Rightarrow H(-f_0) = H(f_0) = \frac{b}{a}$

1. Consider an LTI communication channel.
   Suppose when we put

$$x(t) = 2\cos(2\pi t) + 4\cos(4\pi t) + 6\cos(6\pi t) + 7\cos(8\pi t) + 1$$

into this channel, we get

$$y(t) = \cos(2\pi t) + \cos(4\pi t) + \sin(6\pi t) + 1 + 0\cos(8\pi t)$$

as its output.

a. Let $H(f)$ be the frequency response of the channel that satisfies the above input-output relation.

i. Find $H(2)$.

We have $4\cos(4\pi t) \to \boxed{H(f)} \to \cos(4\pi t)$

$\Rightarrow H(-2) = H(2) = \frac{1}{4}$

ii. Find $H(4)$.

We have $7\cos(8\pi t) \to \boxed{H(f)} \to 0\cos(8\pi t)$

$\Rightarrow H(-4) = H(4) = \frac{0}{7} = 0$

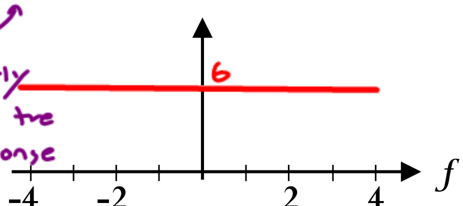
b. Is this channel distortionless?

$|H(2)| \neq |H(4)|$ So, we have amplitude distortion.
The channel is not distortionless.

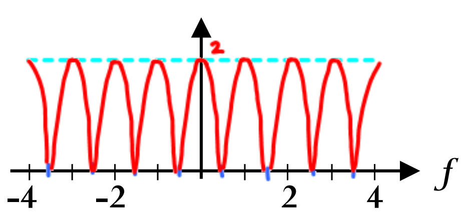
2. [ENRPr] Consider each $g(t)$ defined below.
   Let $G(f)$ be its Fourier transform. Plot $|G(f)|$ from $f = -4$ to $f = 4$ Hz.

(a) $g(t) = 6\delta(t-6)$

Note that this is exactly the same as the impulse response of the distortionless channel.



(b) (optional) $g(t) = \delta(t-6) + \delta(t-5)$



Recall that

$\delta(t) \xrightarrow{\mathcal{F}} 1$ [Ex. 2.19]

From the time-shift property,

$\delta(t-6) \xrightarrow{\mathcal{F}} (e^{-j2\pi(6)t})(1)$

Therefore,

$6\delta(t-6) \xrightarrow{\mathcal{F}} 6\underbrace{e^{-j2\pi(6)t}}_{\text{note that the magnitude of this is 1}}$

See the complete solution on the next page.

Remark: For me, "method 1" is the most intuitive. However, some may find "method 3" to be more straightforward.

(b)

Method 1: When we see two delta functions with equal size in one domain, we suspect cosine in another domain.

First consider

$y(t) = \delta(t-t_0) + \delta(t+t_0)$.

Here, the two delta functions are not centered around $t=0$. Therefore, we expect that shifting property will be applied at some point in the solution.

We know that

$$\cos(2\pi f_0 t) \xrightarrow{\mathcal{F}} \frac{1}{2}\delta(f-f_0) + \frac{1}{2}\delta(f+f_0)$$

By duality theorem

$$\frac{1}{2}\delta(t-f_0) + \frac{1}{2}\delta(t+f_0) \xrightarrow{\mathcal{F}} \cos(2\pi f_0(-f)) = \cos(2\pi f_0 f)$$

$\cos(-x) = \cos(x)$
(cos is an even function)

Let $y(t) = \delta(t+0.5) + \delta(t-0.5)$.

Then, we know that $Y(f) = 2\cos(2\pi(0.5)f)$.

Now, $g(t) = y(t-5.5) = \delta(t-5) + \delta(t-6)$

By time-shift property,

$$G(f) = e^{-j2\pi(5.5)f}\, 2\cos(2\pi(0.5)f) \leftarrow$$ we don't really need this. We only want its magnitude.

$$|G(f)| = |Y(f)| = 2|\cos(2\pi(0.5)f)|$$

Method 2:

Recall, from our lecture on two-path channel (Ex. 3.28),

when we have $h(t) = \beta_1\delta(t-\tau_1) + \beta_2\delta(t-\tau_2)$,

$$|H(f)|^2 = |\beta_1|^2 + |\beta_2|^2 + 2|\beta_1||\beta_2|\cos(2\pi(\tau_2-\tau_1)f + (\underset{\angle\beta_1}{\phi_1} - \underset{\angle\beta_2}{\phi_2})).$$

Here, $\beta_1 = \beta_2 = 1$, $\tau_1 = 5$, $\tau_2 = 6$.

Therefore,

$$|H(f)|^2 = 1^2 + 1^2 + 2\cos(2\pi f) = 2 + 2\cos(2\pi f)$$

$$= 4\left(\frac{1+\cos(2\pi f)}{2}\right) = 4\left(\cos(2\pi(0.5)f)\right)^2$$

Recall, from Ex. 2.4, that $\cos^2(x) = \frac{1}{2}(1+\cos(2x))$

$$|H(f)| = 2|\cos(2\pi(0.5)f)|$$

Method 3:

From $\delta(t-t_0) \xrightarrow{\mathcal{F}} e^{-j2\pi t_0 f}$,

we have

This is factored out to create the ± 0.5 term.

$$G(f) = e^{-j2\pi 5f} + e^{-j2\pi 6f} = \underbrace{e^{-j2\pi(5.5)f}}\left(e^{+j2\pi(0.5)f} + e^{-j2\pi(0.5)f}\right)$$

$$= e^{-j2\pi(5.5)f}\, 2\cos(2\pi(0.5)f).$$

$$|G(f)| = 2|\cos(2\pi\underbrace{(0.5)}f)|$$

"freq" = 0.5

"period" = $\frac{1}{0.5}$ = 2