ECS 332: In-Class Exercise # 5

Date: **11** / **09** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

In this problem, we have three "devices".

- $\boxed{(\cdot)^2}$ is a "square" device. As the name suggests, its output is created by squaring its input in the **time** domain.
- $\boxed{H_1(f)}$ is an LTI device whose **frequency response** is $H_1(f) = \begin{cases} 1, & |f| < 200, \\ 0, & \text{otherwise.} \end{cases}$
- $\boxed{H_2(f)}$ is an LTI device whose **frequency response** is $H_2(f) = \begin{cases} 1, & |f| > 200, \\ 0, & \text{otherwise.} \end{cases}$

Recall that $e^{j2\pi f_0 t} \to \boxed{H(f)} \to H(f_0)e^{j2\pi f_0 t}$

Find the output $y(t)$ for each of the systems below.

(a) $x(t) = \cos(\underbrace{300\pi}_{2\pi f_0} t) \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$ $\quad 2\pi f_0 t \Rightarrow f_0 = 150$ Hz

$= \frac{1}{2}e^{j2\pi(150)t} + \frac{1}{2}e^{j2\pi(-150)t}$

$H_1(\pm 150) = 1$ because $|\pm 150| < 200$.

$y(t) = \underbrace{H_1(150)}_{1}\frac{1}{2}e^{j2\pi(150)t} + \underbrace{H_1(-150)}_{1}\frac{1}{2}e^{j2\pi(-150)t} = \frac{1}{2}e^{j2\pi(150)t} + \frac{1}{2}e^{j2\pi(-150)t} = \cos(300\pi t).$

(b) $x(t) = \cos(300\pi t) \longrightarrow \boxed{H_2(f)} \longrightarrow y(t)$ $\quad H_2(\pm 150) = 0$ because $|\pm 150| \not> 200$.

$y(t) = \underbrace{H_2(150)}_{0}\frac{1}{2}e^{j2\pi(150)t} + \underbrace{H_2(-150)}_{0}\frac{1}{2}e^{j2\pi(-150)t} = 0.$

(c) $x(t) = \cos(\underbrace{300\pi t}_{\theta}) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \boxed{H_1(f)} \longrightarrow y(t)$

$H_1(0) = 1$ because $|0| < 200$

$H_1(\pm 300) = 0$ because $|\pm 300| \not< 200$

$x^2(t) = \left(\frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta}\right)^2 = \frac{1}{4}\left(e^{j(2\theta)} + 2 + e^{j(-2\theta)}\right)$

$y(t) = \frac{1}{4}\left(\underbrace{H_1(300)}_{0}e^{j2\pi(300)t} + 2\underbrace{H_1(0)}_{1} + \underbrace{H_1(-300)}_{0}e^{j2\pi(-300)t}\right) = \frac{1}{4}\times 2\times 1 = \frac{1}{2}.$

(d) $x(t) = \cos(300\pi t) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \boxed{H_2(f)} \longrightarrow y(t)$

$H_2(0) = 0$ because $0 \not> 200$

$H_2(\pm 300) = 1$ because $|\pm 300| > 200$

$y(t) = \frac{1}{4}\left(\underbrace{H_2(300)}_{1}e^{j2\pi(300)t} + 2\underbrace{H_2(0)}_{0} + \underbrace{H_2(-300)}_{1}e^{j2\pi(-300)t}\right) = \frac{1}{4}\left(e^{j600\pi t} + e^{-j600\pi t}\right) = \frac{1}{2}\cos(600\pi t).$

(e) $x(t) = \cos(300\pi t) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \boxed{H_1(f)} \xrightarrow{v(t)} \boxed{H_2(f)} \longrightarrow y(t)$

From part (c), $v(t) = \frac{1}{2}$.

$y(t) = H_2(0)\frac{1}{2} = 0 \times \frac{1}{2} = 0.$

(f) $x(t) = \cos(300\pi t) \longrightarrow \boxed{H_1(f)} \longrightarrow \boxed{(\cdot)^2} \longrightarrow \boxed{H_2(f)} \longrightarrow y(t)$

From part (a), we know that we still have $\cos(300\pi t)$ here.

From part (d), we have $y(t) = \frac{1}{2}\cos(600\pi t)$

ECS 332: In-Class Exercise # 5 Alternative solution

Date: _ _ / _ _ / 2018			
Name	ID (last 3 digits)		

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.**
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- $\boxed{H_1(f)}$ is an LTI device whose **frequency response** is $H_1(f) = \begin{cases} 1, & |f| < 200, \\ 0, & \text{otherwise.} \end{cases}$
- $\boxed{H_2(f)}$ is an LTI device whose **frequency response** is $H_2(f) = \begin{cases} 1, & |f| > 200, \\ 0, & \text{otherwise.} \end{cases}$

Recall that

$e^{j2\pi f_0 t} \longrightarrow \boxed{H(f)} \longrightarrow H(f_0) e^{j2\pi f_0 t}$

$\cos(2\pi f_0 t) \longrightarrow \boxed{H(f)} \longrightarrow \frac{1}{2}H(f_0)e^{j2\pi f_0 t} + \frac{1}{2}H(-f_0)e^{-j2\pi f_0 t}$

$= H(f_0)\cos(2\pi f_0 t)$

if $H(f_0) = H(-f_0)$

For this problem, $H_1(-f) = H_1(f)$ and $H_2(-f) = H_2(f)$ at all freq. Therefore, the result above can be applied.

Find the output $y(t)$ for each of the systems below.

$\underbrace{2\pi f_0}\, t \Rightarrow f_0 = 150$ Hz

(a) $x(t) = \cos(300\pi t) \longrightarrow \boxed{H_1(f)} \longrightarrow y(t)$

$H_1(\pm 150) = 1$ because $|\pm 150| < 200$.

Therefore, $x(t)$ will pass through $H_1(f)$ unchanged.

$\Rightarrow y(t) = x(t) = \cos(300\pi t)$

(b) $x(t) = \cos(300\pi t) \longrightarrow \boxed{H_2(f)} \longrightarrow y(t)$

$H_2(\pm 150) = 0$ because $|\pm 150| \not> 200$.

Therefore, $x(t)$ can not pass through $H_2(f)$.

$\Rightarrow y(t) \equiv 0$

(c) $x(t) = \cos(300\pi t) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \boxed{H_1(f)} \longrightarrow y(t)$

$(\cos(2\pi(150)t))^2 = \frac{1}{2} + \frac{1}{2}\cos(2\pi(300)t)$.

$H_1(0) = 1$ because $|0| < 200$

$H_1(\pm 300) = 0$ because $|\pm 300| \not< 200$

Therefore, only the DC component of $x^2(t)$ can pass through $H_1(t)$.

$\Rightarrow y(t) \equiv 1/2$

(d) $x(t) = \cos(300\pi t) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \boxed{H_2(f)} \longrightarrow y(t)$

$H_2(0) = 0$ because $0 \not> 200$

$H_2(\pm 300) = 1$ because $|\pm 300| > 200$

Therefore, only the components at ± 300 Hz of $x^2(t)$ can pass through $H_2(t)$.

$\Rightarrow y(t) = \frac{1}{2}\cos(2\pi(300)t) = \frac{1}{2}\cos(600\pi t)$

(e) $x(t) = \cos(300\pi t) \longrightarrow \boxed{(\cdot)^2} \xrightarrow{x^2(t)} \underbrace{\boxed{H_1(f)} \longrightarrow \boxed{H_2(f)}} \longrightarrow y(t)$

$H_1(f)H_2(f) \equiv 0$ for all f

Therefore, no signal can pass through this system. $\Rightarrow y(t) \equiv 0$

(f) $x(t) = \cos(300\pi t) \longrightarrow \boxed{H_1(f)} \longrightarrow \boxed{(\cdot)^2} \longrightarrow \boxed{H_2(f)} \longrightarrow y(t)$

From part (a), we know that we still have $\cos(300\pi t)$ here.

From part (d), we have $y(t) = \frac{1}{2}\cos(600\pi t)$