Name ________________________ ID3 _ _ _

ECS 332: Principles of Communications **2018/1**

HW 4 — Due: Not Due

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Problem 1. Consider the DSB-SC modem with no channel impairment shown in Figure 4.1. Suppose that the message is band-limited to $B = 3$ kHz and that $f_c = 100$ kHz.

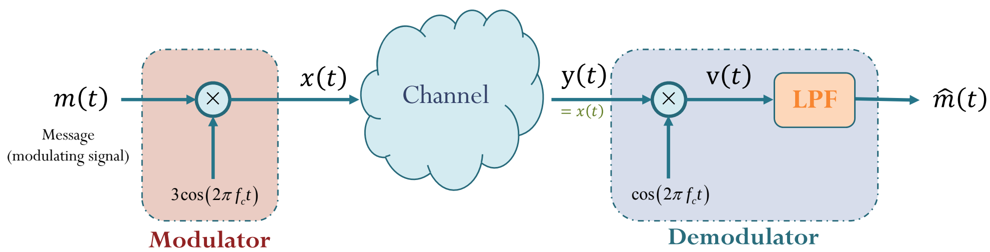

Figure 4.1: DSB-SC modem with no channel impairment

(a) Specify the frequency response $H_{LP}(f)$ of the LPF so that $\hat{m}(t) = m(t)$.

(b) Suppose the impluse response $h_{LP}(t)$ of the LPF is of the form $\alpha\,\text{sinc}(\beta t)$. Find the constants α and β such that $\hat{m}(t) = m(t)$.

Problem 2. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 4.2. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

(a) Find the energy in each signal.

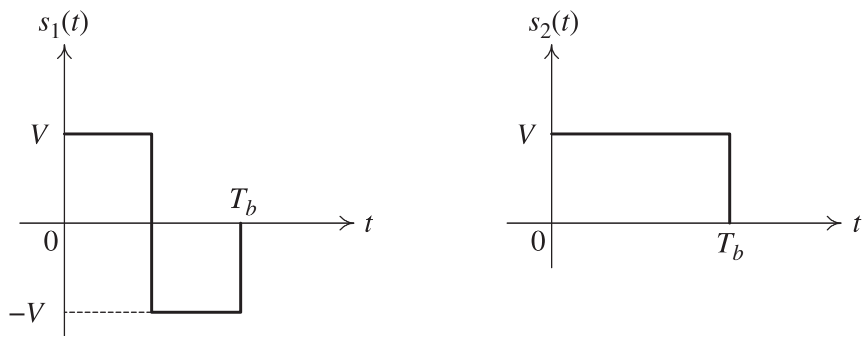

Figure 4.2: Signal set for Question 2

(b) Are they energy signals?

(c) Are they power signals?

(d) Find the (average) power in each signal.

(e) Are the two signals $s_1(t)$ and $s_2(t)$ orthogonal?

Problem 3. (Power Calculation) For each of the following signals $g(t)$, find (i) its corresponding power $P_g = \left\langle |g(t)|^2 \right\rangle$, (ii) the power $P_x = \left\langle |x(t)|^2 \right\rangle$ of $x(t) = g(t)\cos(10t)$, and (iii) the power $P_y = \left\langle |y(t)|^2 \right\rangle$ of $y(t) = g(t)\cos(50t)$

(a) $g(t) = 3\cos(10t + 30°)$.

(b) $g(t) = 3\cos(10t + 30^\circ) + 4\cos(10t + 120^\circ)$. (Hint: First, use phasor form to combine the two components into one sinusoid.)

(c) $g(t) = 3\cos(10t) + 3\cos(10t + 120°) + 3\cos(10t + 240°)$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 4. This question starts with a ***square-modulator*** for DSB-SC . Then, the use of the square-operation block is further explored on the receiver side of the system. [Doerschuk, 2008, Cornell ECE 320]

(a) Let $x(t) = A_c m(t)$ where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Consider the block diagram shown in Figure 4.3.

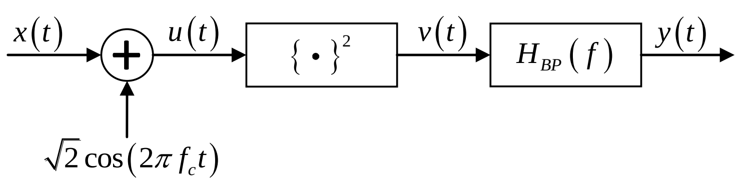

Figure 4.3: Block diagram for Problem 4a

Assume $f_c \gg B$ and

$$H_{BP}(f) = \begin{cases} 1, & |f - f_c| \le B \\ 1, & |f + f_c| \le B \\ 0, & \text{otherwise.} \end{cases}$$

The block labeled "$\{\cdot\}^2$" has output $v(t)$ that is the square of its input $u(t)$:

$$v(t) = u^2(t).$$

Find $y(t)$.

(b) The block diagram in part (a) provides a nice implementation of a modulator because it may be easier to build a squarer than to build a multiplier. Based on the successful use of a squaring operation in the modulator, we decide to use the same squaring operation in the demodulator. Let

$$x\left(t\right) = A_c m\left(t\right)\sqrt{2}\cos\left(2\pi f_c t\right)$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 4.4.

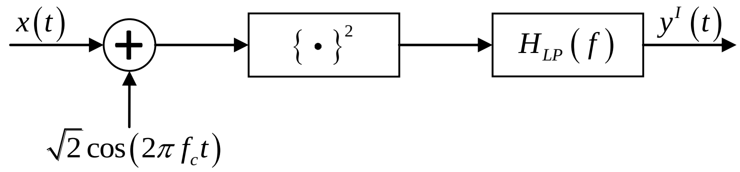

Figure 4.4: Block diagram for Problem 4b

Use

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^I(t)$. Does this block diagram work as a demodulator; that is, is $y^I(t)$ proportional to $m(t)$?

(c) Due to the failure in part (b), we have to think hard and it seems natural to consider also the block diagram with cos replaced by sin. Let

$$x(t) = A_c m(t)\sqrt{2}\cos(2\pi f_c t)$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$ as in part (b). Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 4.5.

As in part (b), use

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

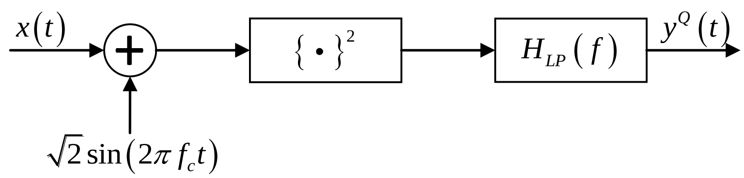

Figure 4.5: Block diagram for Problem 4c

Find $y^Q(t)$.

(d) Use the results from parts (b) and (c). Draw a block diagram of a ***successful*** DSB-SC demodulator using squaring operations instead of multipliers.

Problem 5 (Cube modulator)**.** Consider the block diagram shown in Figure 4.6 where "$\{\cdot\}^3$" indicates a device whose output is the cube of its input.

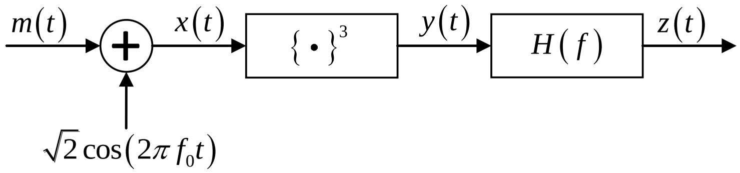

Figure 4.6: Block diagram for Problem 5. Note the use of f_0 instead of f_c.

Let $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ be bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$.

(a) Plot an $H(f)$ that gives $z(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$. What is the gain in $H(f)$? What is the value of f_c? Notice that the frequency of the cosine is f_0 not f_c. You are supposed to determine f_c in terms of f_0.

(b) Let $M(f)$ be

$$M\left(f\right)=\begin{cases}1, & \left|f\right|\leq B\\ 0, & \text{otherwise.}\end{cases}$$

(i) Plot $X(f)$.

(ii) Plot $Y(f)$. Hint:

$$M\left(f\right)*M\left(f\right)=\begin{cases}2B-\left|f\right|, & \left|f\right|\leq 2B\\ 0, & \text{otherwise.}\end{cases}$$

Do not attempt to make an accurate plot or calculation for the Fourier transform of $m^3(t)$.

(iii) For your filter of part (a), plot $z(t)$.

[Doerschuk, 2008, Cornell ECE 320]

Problem 6. Consider a signal $g(t)$. Recall that $|G(f)|^2$ is called the **energy spectral density** of $g(t)$. Integrating the energy spectral density over all frequency gives the signal's total energy. Furthermore, the energy contained in the frequency band I can be found from the integral $\int_I |G(f)|^2 df$ where the integration is over the frequencies in band I. In particular, if the band is simply an interval of frequency from f_1 to f_2, then the energy contained in this band is given by

$$\int_{f_1}^{f_2} |G(f)|^2 df. \tag{4.1}$$

In this problem, assume

$$g(t) = 1[-1 \leq t \leq 1].$$

(a) Find the (total) energy of $g(t)$.

(b) Figure 4.7 define the main lobe of a sinc pulse. It is well-known that the main lobe of the sinc function contains about 90% of its total energy. Check this fact by first

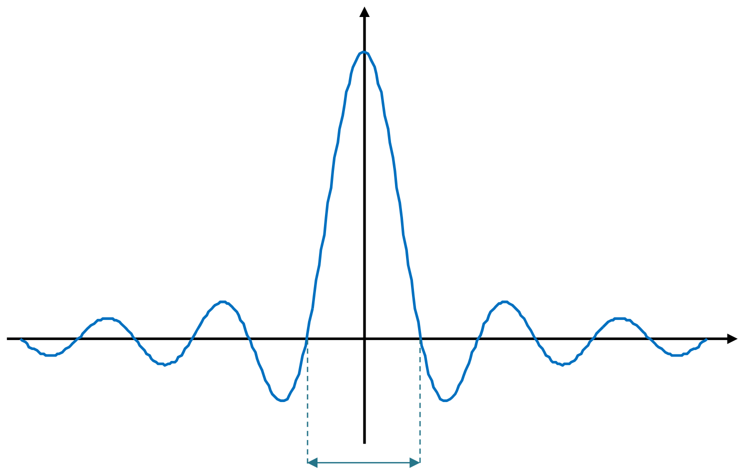

Figure 4.7: Main lobe of a sinc pulse

computing the energy contained in the frequency band occupied by the main lobe and then compare with your answer from part (a).

Hint: Find the zeros of the main lope. This give f_1 and f_2. Now, we can apply (4.1). `MATLAB` or similar tools can then be used to numerically evaluate the integral.

(c) Suppose we want to include more energy by considering wider frequency band. Let this band be the interval $I = [-f_0, f_0]$. Find the minimum value of f_0 that allows the band to capture at least 99% of the total energy in $g(t)$.