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ECS 332: Principles of Communications 2017/1

HW 6 — Due: Nov 3, 4 PM **Solution**

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Instructions

(a) This assignment has 6 pages.

(b) (1 pt) Work and write your answers **directly on these provided sheets** (not on other blank sheet(s) of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Find the frequency of the signal $g(t) = 3\sqrt{2}\cos\left(12t^3 + t^2\right)$

(a) at time $t = 0$

The instantaneous freq. of a signal of the form $g(t) = A\cos(\theta(t))$ is

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\theta(t).$$

Here, $\theta(t) = 12t^3 + t^2$. Therefore, $f(t) = \frac{1}{2\pi}\left(12\times 3t^2 + 2t\right) = \frac{1}{\pi}\left(18t^2 + t\right)$

$$f(0) = \frac{1}{\pi}\left(18\times 0^2 + 0\right) = 0 \text{ [Hz]}.$$

(b) at time $t = 2$

$$f(2) = \frac{1}{\pi}\left(18\times 2^2 + 2\right) = \frac{74}{\pi} \approx 23.55 \text{ [Hz]}.$$

Problem 2. Recall that, in QAM system, the transmitted signal is of the form

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

We want to express x_{QAM} in the form

$$x_{\text{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \phi(t)),$$

where $E(t) \geq 0$ and $\phi(t) \in (-180°, 180°]$. (This shows that QAM can be expressed as a combination of amplitude modulation and phase modulation.)

Consider $m_1(t)$ and $m_2(t)$ plotted in Figure 6.1.
Draw the corresponding $E(t)$ and $\phi(t)$.

In lecture, we have shown that, at time t,

$E(t) = |m_1(t) - jm_2(t)|$

and

$\phi(t) = \angle(m_1(t) - jm_2(t))$.

Therefore, we simply need to find $m_1(t) - jm_2(t)$ and change it into the corresponding polar form.

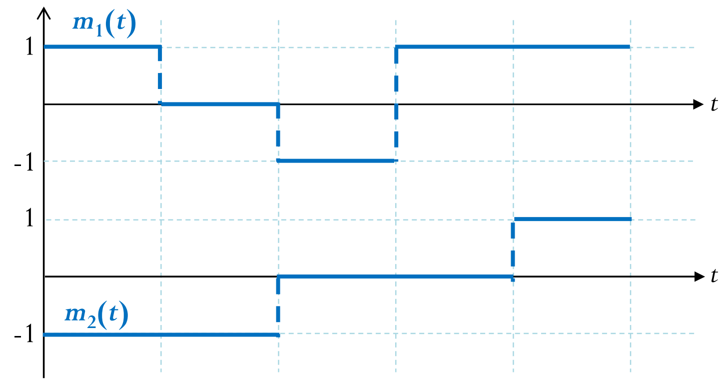


Figure 6.1: $m_1(t)$ and $m_2(t)$ for Problem 2

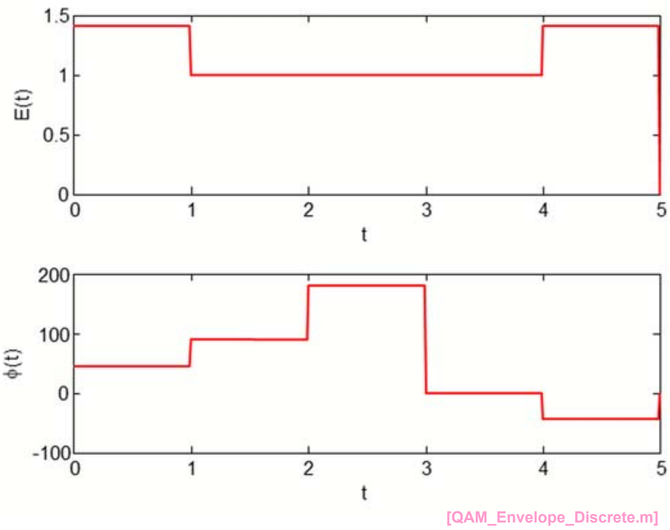
Note that once we get the value of $m_1 - jm_2$, the conversion to polar form can be done easily with the help of your calculator. You should have done similar conversion in circuit analysis class.

Below, we list the values of $m_1(t)$ and $m_2(t)$ during each time interval:

convert this to polar form: $E \angle \phi$

m_1	m_2	$m_1 - jm_2$	E	ϕ
1	-1	$1 + j$	$\sqrt{1^2 + 1^2} = \sqrt{2}$	45°
0	-1	j	1	90°
-1	0	-1	1	180°
1	0	1	1	0°
1	1	$1 - j$	$\sqrt{1^2 + (-1)^2} = \sqrt{2}$	-45°

Here, the conversion is visually obvious; so, in fact, we can do the conversion without the help of a calculator.



Problem 3. Consider the message $m(t)$ along with the carrier signal $\cos(2\pi f_c t + \phi)$ plotted in Figure 6.2.

$m_p = 40$

$-m_p = -40$

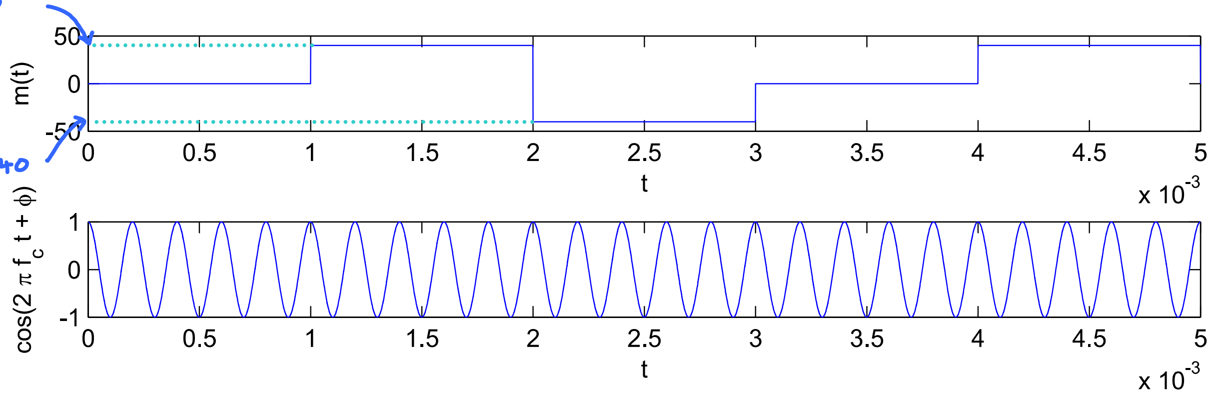


Figure 6.2: The message and the carrier signals for Problem 3.

(a) Find the carrier frequency f_c from the plot. (Hint: It is an integer.)

During time $t=0$ to $t=10^{-3}$, there are 5 cycles of $\cos(2\pi f_c t + \phi)$.

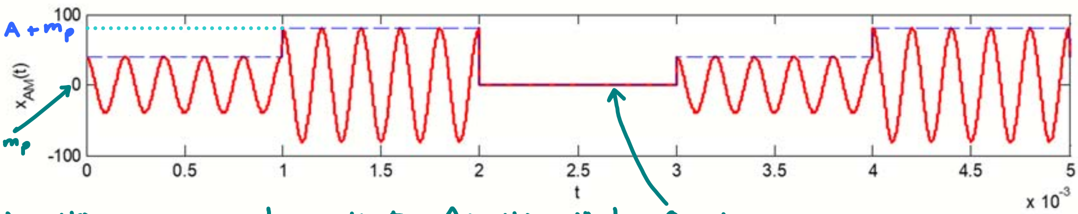
Therefore, its frequency is $f_c = \frac{5}{10^{-3}} = 5$ kHz.

(b) Sketch the following signals. Make sure that (the unspecified parameter(s) are selected such that) the important "features" of the graphs can be seen clearly.

(i) $x_{\mathrm{AM}}(t) = (A + m(t))\cos(2\pi f_c t + \phi)$ whose modulation index $\mu = 100\%$. $= \frac{A}{m_p}$ $\Rightarrow A = m_p = 40$

$80 = 40 + 40 = A + m_p$

$0 = 40 - 40 = A - m_p$



With $\min\limits_t m(t) = -m_p$, we know that $A + m(t)$ will be 0 when $m(t)$ is having its minimum value.

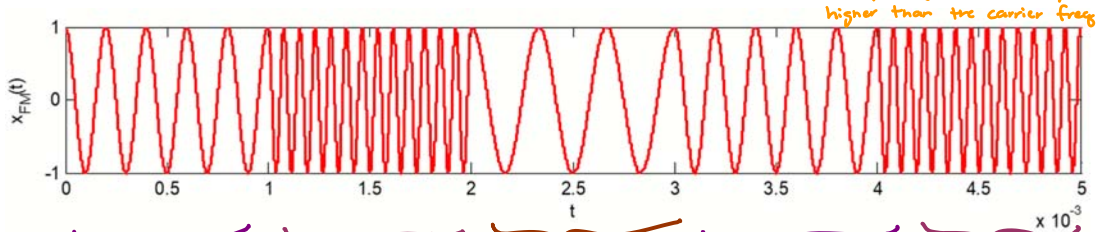
(ii) $x_{\mathrm{FM}}(t) = A\cos\left(2\pi f_c t + \phi + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right)$

Case 1: When $m(t) = 0$, we should have $f(t) = f_c + 0 = f_c$.

Case 2: When $m(t)$ is at its "high" value, the inst. freq. $f(t)$ of $x_{FM}(t)$ should also be at its "high" value.

The "high" value of $m(t)$ is > 0. So, the corresponding $f(t) = f_c + k_f m(t)$ should be higher than the carrier freq. f_c.

- You may assume $m(t) = 0$ for $t < 0$.



1 2 3 1 2

Case 3: When $m(t)$ is at its "low" value, the inst. freq. $f(t)$ of $x_{FM}(t)$ should also be at it "low" value.

Jump location t [ms]	Δm	$\Delta\phi = k_p \Delta m$
1	$+m_p$	π
2	$-2m_p$	-2π
3	$+m_p$	π
4	$+m_p$	π

$k_p = \pi/m_p$

Case ②: When a sinusoid is advanced by "-2π", we simply get the same waveform: $\cos(\beta - 2\pi) = \cos(\beta)$.

Case ①: When a sinusoid is advanced by π, we simply get its "−": $\cos(\beta + \pi) = -\cos(\beta)$.

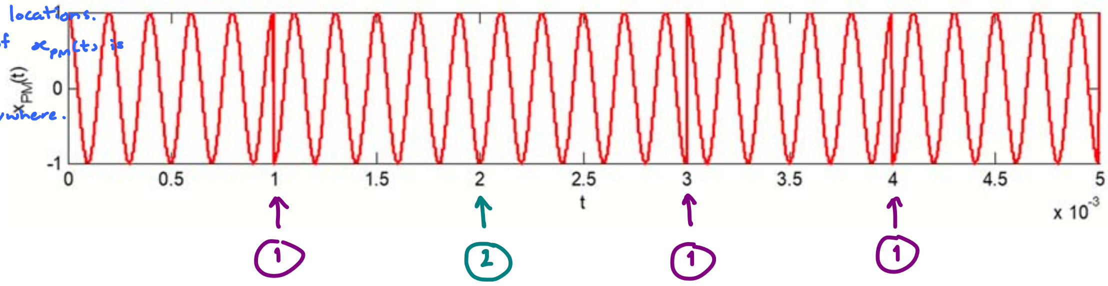
Note that $m(t)$ is piecewise-constant. Its values jump at various places but there is no place that $m(t)$ changes gradually.

The derivative of $m(t)$ is 0 almost everywhere except at the jump locations.

Recall that the inst. freq. of $x_{PM}(t)$ is $f(t) = f_c + \frac{k_p}{2\pi}\dot{m}(t)$.

Therefore, $f(t) = f_c$ almost everywhere.

(iii) $x_{\mathrm{PM}}(t) = A\cos(2\pi f_c t + \phi + k_p m(t))$ with $k_p = \frac{\pi}{m_p}$.



Problem 4. Consider the FM transmitted signal

$$x_{\mathrm{FM}}(t) = A\cos\left(2\pi f_c t + \phi + 2\pi k_f \int_{-\infty}^{t} m(\tau)d\tau\right),$$

where $f_c = 5$ [kHz], $A = 1$, and $k_f = 75$. The message $m(t)$ is shown in Figure 6.3.

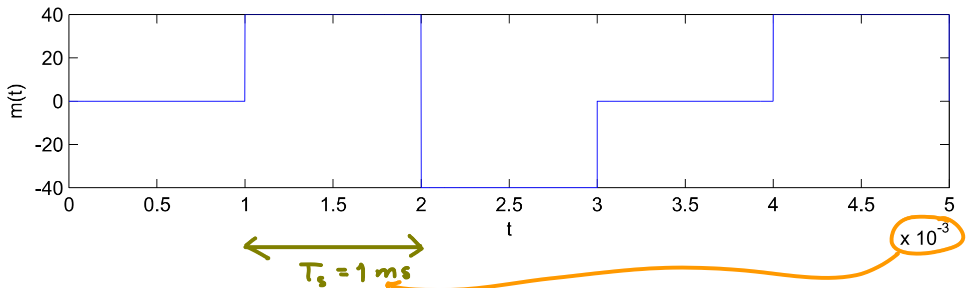


Figure 6.3: The message $m(t)$ for Problem 4

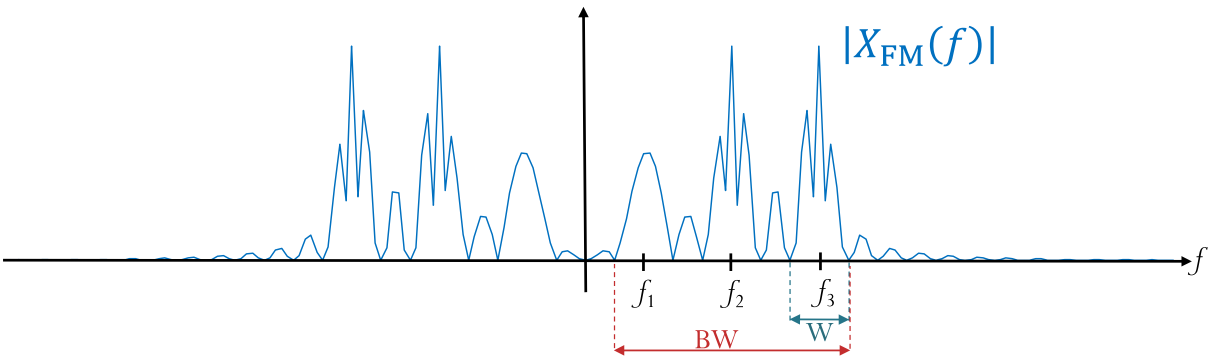


Figure 6.4: The magnitude spectrum $|X_{\mathrm{FM}}(f)|$ for Problem 4

The magnitude spectrum $|X_{\mathrm{FM}}(f)|$ is plotted in Figure 6.4.

(a) Find the values of f_1, f_2, and f_3. First, observe that m(t) takes only 3 values: -40, 0, 40.
For FM, we know that $f(t) = f_c + k_f m(t)$. Therefore, here, the instantaneous freq. of the transmitted signal will take only three corresponding values. Plugging-in the possible values of m(t) (from the lowest one to the highest one), we get

$$f_1 = f_c + k_f(-40) = 5\times10^3 + 75(-40) = 5000 - 3000 = 2 \text{ kHz}$$
$$f_2 = f_c + k_f(0) = f_c = 5 \text{ kHz}$$
$$f_3 = f_c + k_f(40) = 5\times10^3 + 75(40) = 5000 + 3000 = 8 \text{ kHz}$$

Each portion of m(t) creates a cosine pulse in the transmitted signal. Each cosine pulse corresponds to two sinc functions in the freq. domain: one at the freq. of the pulse and another at its negative.

(b) Find the width W in Figure 6.4.
The "width" of each of the sinc pulse in the freq. domain is $W = \frac{2}{T_s}$.

Here, from the plot of $m(t)$, we have $T_s = 1$ ms.
Therefore, $W = \frac{2}{1\times10^{-3}} = 2$ kHz.

(c) Find the occupied bandwidth denoted by BW in Figure 6.4.

$$BW = \frac{1}{T_s} + (\underset{\uparrow\, f_{max}}{f_3} - \underset{\uparrow\, f_{min}}{f_1}) + \frac{1}{T_s} = (8-2) + 2 = 8 \text{ kHz}$$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 5. Recall that, in QAM system, the transmitted signal is of the form

$$x_{\mathrm{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

In class, we have shown that

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\cos(2\pi f_c t)\right\} = m_1(t).$$

Give a similar proof to show that

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\sin(2\pi f_c t)\right\} = m_2(t).$$

Let $v(t) = x_{QAM}(t)\sqrt{2}\sin(2\pi f_c t)$

$$= m_1(t)\underbrace{2\cos(2\pi f_c t)\sin(2\pi f_c t)} + m_2(t)\underbrace{2\sin(2\pi f_c t)\sin(2\pi f_c t)}$$

$$2\cos x \sin x = \cancel{2}\left(\frac{e^{jx}+e^{-jx}}{\cancel{2}}\right)\left(\frac{e^{jx}-e^{-jx}}{2j}\right) = \frac{1}{2j}\left(e^{j2x} - e^{-j2x}\right) = \sin(2x)$$

$$\sin^2 x = \left(\frac{e^{jx}-e^{-jx}}{2j}\right)^2 = \frac{e^{2jx}+e^{-2jx}-2}{-4} = \frac{1}{2}\left(1-\cos(2x)\right)$$

$$= m_1(t)\overbrace{\sin(2\pi(2f_c)t)} + m_2(t)\overbrace{(1-\cos(2\pi(2f_c)t)}$$

$v(t) = m_2(t) + \underbrace{m_1(t)\sin(2\pi(2f_c)t)}_{\text{the spectrum is centered around } \pm 2f_c} - \underbrace{m_2(t)\cos(2\pi(2f_c)t)}_{\text{the spectrum is centered around } \pm 2f_c}$

$\mathrm{LPF}\{v(t)\} = m_2(t) + 0 + 0 = m_2(t)$

Assumption: If $m_1(t)$ is band limited to B_1
$m_2(t)$ is band limited to B_2,
we need $f_c > B_1$ and $f_c > B_2$

Problem 6. In ***quadrature amplitude modulation*** (***QAM***) or ***quadrature multiplexing***, two baseband signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the following QAM signal:

$$x_{\mathrm{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t).$$

An error in the phase or the frequency of the carrier at the demodulator in QAM will result in loss and interference between the two channels (cochannel interference).

In this problem, show that

(a): $\mathrm{LPF}\left\{\overbrace{x_{\mathrm{QAM}}(t)\sqrt{2}\cos((\omega_c+\Delta\omega)t+\delta)}^{v_1(t)}\right\} = m_1(t)\cos((\Delta\omega)t+\delta) - m_2(t)\sin((\Delta\omega)t+\delta)$

(b): $\mathrm{LPF}\left\{\underbrace{x_{\mathrm{QAM}}(t)\sqrt{2}\sin((\omega_c+\Delta\omega)t+\delta)}_{v_2(t)}\right\} = m_1(t)\sin((\Delta\omega)t+\delta) + m_2(t)\cos((\Delta\omega)t+\delta).$

(a) Let $\hat{m}_1(t) = \mathrm{LPF}\{v_1(t)\}$.

Then, by the product-to-sum formulas, See Remark #1.

$$v_1(t) = m_1(t)\cos(\underbrace{(2\omega_c+\Delta\omega)t+\delta}_{\text{LPF}\to 0}) + m_1(t)\cos((\Delta\omega)t+\delta)$$
$$+\, m_2(t)\sin(\underbrace{(2\omega_c+\Delta\omega)t+\delta}_{\text{LPF}\to 0}) + m_2(t)\sin(-(\Delta\omega)t-\delta)$$

$$\hat{m}_1(t) = m_1(t)\cos((\Delta\omega)t+\delta) - m_2(t)\sin((\Delta\omega)t+\delta)$$

(b) Let $\hat{m}_2(t) = \text{LPF}\{v_2(t)\}$.

Then, by the product-to-sum formulas, See Remark #1.

$$v_2(t) = m_1(t)\sin\big(\underbrace{\cancel{(2\omega_c+\Delta\omega)t+\delta}}_{\text{LPF}\to 0}\big) + m_1(t)\sin\big((\Delta\omega)t+\delta\big)$$
$$+ m_2(t)\cos\big((\Delta\omega)t+\delta\big) - m_2(t)\cos\big(\underbrace{\cancel{(2\omega_c+\Delta\omega)t+\delta}}_{\text{LPF}\to 0}\big)$$

$$\hat{m}_2(t) = m_1(t)\sin\big((\Delta\omega)t+\delta\big) + m_2(t)\cos\big((\Delta\omega)t+\delta\big)$$

Problem 7. As in Problem 2, in QAM system, the transmitted signal is of the form

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

We want to express x_{QAM} in the form

$$x_{\text{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \phi(t)),$$

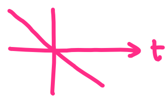
where $E(t) \geq 0$ and $\phi(t) \in (-180°, 180°]$.

In each part below, we consider different examples of the messages $m_1(t)$ and $m_2(t)$.

(a) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = \sin(2\pi Bt)$ where $0 < B \ll f_c$. Find $E(t)$ and $\phi(t)$. Hint: $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha - \beta)$ ← See Remark #2.

$$x_{QAM}(t) = \sqrt{2}\cos(2\pi Bt)\cos(2\pi f_c t) + \sqrt{2}\sin(2\pi Bt)\sin(2\pi f_c t)$$
$$= \sqrt{2}\cos(2\pi f_c t - 2\pi Bt)$$

$$E(t) = \sqrt{2} \qquad \underbrace{-2\pi Bt}_{= \phi(t)\text{ ? No!}}$$



We can't stop here because the question needs $\phi(t) \in (-180°, 180°]$.

It is clear that when t is large, "$-2\pi Bt$" will exceed $-180°$.

We may refer to "$-2\pi Bt$" as the "unwrapped phase".

Adding/subtracting appropriate multiple of 360° will bring "$-2\pi Bt$" into the $(-180°, 180°]$ range.

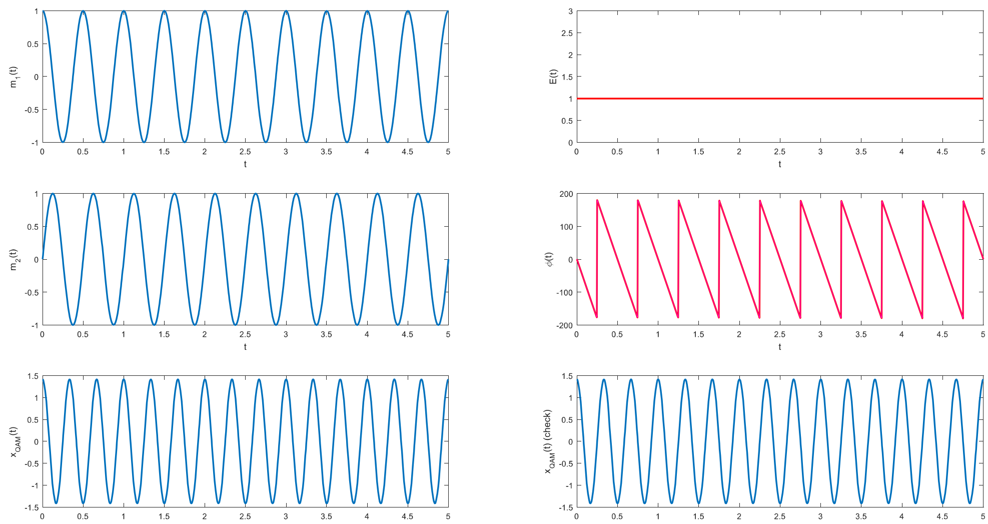
Mathematically, this could be done via

$$\phi(t) = \big((-2\pi Bt + 180°) \bmod 360°\big) - 180°.$$

We may refer to this as the "wrapped phase".

(b) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = 2\sin(2\pi Bt)$. Let $f_c = 5$ and $B = 2$. Use `MATLAB` to plot the corresponding $E(t)$ and $\phi(t)$ from $t = 0$ to $t = 5$ [sec]. (Hint: the function `angle` or `atan2` will be helpful here.)

The plot for part (a):



(b) We can apply the command "abs" and "angle" to $m_1(t) - j m_2(t)$ to find $E(t)$ and $\phi(t)$ respectively.
Alternatively, we can find $E(t)$ from

$$E(t) = \sqrt{m_1^2(t) + m_2^2(t)} = \sqrt{\cos^2(2\pi Bt) + (2\sin(2\pi Bt))^2} = \sqrt{\cos^2 A + 4\sin^2 A}$$

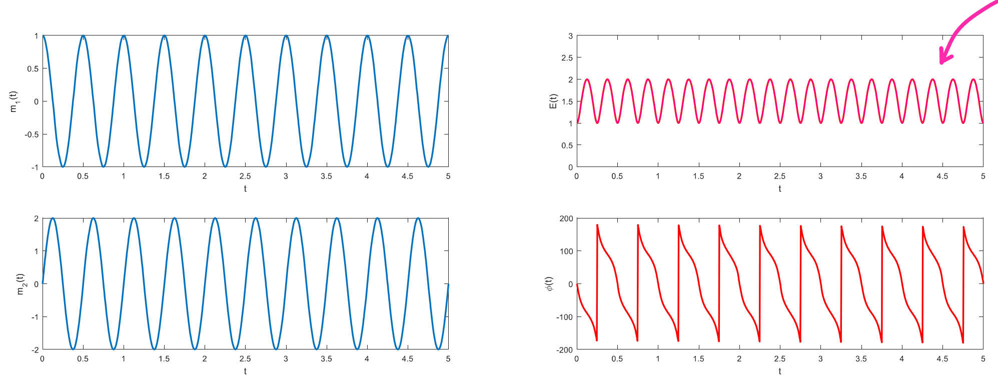
$A = 2\pi Bt$

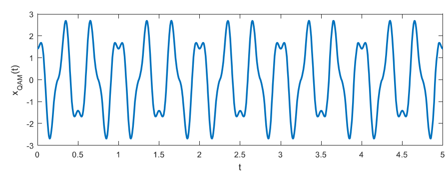
$$= \sqrt{\cos^2 A + 4(1-\cos^2 A)} = \sqrt{4 - 3\cos^2 A}$$

$$= \sqrt{4 - 3\left(\tfrac{1}{2}(1+\cos(2A))\right)} = \sqrt{\tfrac{5}{2} - \tfrac{3}{2}\cos(2A)} = \sqrt{\tfrac{5}{2} - \tfrac{3}{2}\cos(2\pi(2B)t)}$$

The plot for part (b):

Note that the freq. is doubled.





The command atan2$(m_1(t), -m_2(t))$ can also be used to find $\phi(t)$.

Caution: Because the question want $\phi(t) \in [-180°, 180°]$, don't forget to convert the unit from "radians" to "degrees".

$x(t) \xrightarrow{\mathcal{F}} X(f).$

Problem 8. Consider a complex-valued signal $x(t)$ whose Fourier transform is $X(f)$.

(a) Find and simplify the Fourier transform of $x^*(t)$.

Let $y(t) = x^*(t)$. We want to find $Y(f)$.

First, recall that $X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi f t} dt$.

Hence, $Y(f) = \int_{-\infty}^{\infty} x^*(t) e^{-j2\pi f t} dt = \Big(\underbrace{\int_{-\infty}^{\infty} x(t) e^{-j2\pi(-f)t} dt}_{X(-f)} \Big)^* = (X(-f))^* = X^*(-f)$

(b) Find and simplify the Fourier transform of $\text{Re}\{x(t)\}$.

- Hint: $x(t) + x^*(t) = ?$

Let $y(t) = \text{Re}\{x(t)\}$.

From the hint, we first note that $x(t) + x^*(t) = 2\text{Re}\{x(t)\}$.

Hence, $y(t) = \text{Re}\{x(t)\} = \frac{1}{2}\left(x(t) + x^*(t)\right)$ and

$Y(f) = \frac{1}{2}\left(X(f) + \mathcal{F}\{x^*(t)\}\right) = \frac{1}{2}\left(X(f) + X^*(-f)\right)$ See Remark #3.

From part (a)

Problem 9. Consider a (complex-valued) baseband signal $x_b(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} X_b(f)$ which is band-limited to B, i.e., $|X_b(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

(a) The passband signal $x_p(t)$ is given by

$$x_p(t) = \sqrt{2}\,\text{Re}\left\{e^{j2\pi f_c t} x_b(t)\right\}.$$

Find and simplify the Fourier transform of $x_p(t)$.

By the freq.-shift property of Fourier transform,

$\underbrace{e^{j2\pi f_c t} x_b(t)}_{\text{call this } g(t).} \xrightarrow{\mathcal{F}} X_b(f - f_c)$

Then, $G(f) = X_b(f - f_c)$ and $x_p(t) = \sqrt{2}\,\text{Re}\{g(t)\}$.

Recall, from the previous problem that $\text{Re}\{g(t)\} \xrightarrow{\mathcal{F}} \frac{1}{2}\left(G(f) + G^*(-f)\right)$.

Hence, $X_p(f) = \sqrt{2} \times \frac{1}{2}\left(G(f) + G^*(-f)\right) = \frac{1}{\sqrt{2}}\left(X_b(f - f_c) + X_b^*(-f - f_c)\right)$

(b) Find and simplify

$$\mathrm{LPF}\left\{\sqrt{2}\left(\underbrace{\sqrt{2}\,\mathrm{Re}\left\{e^{j2\pi f_c t}x_b(t)\right\}}_{x_p(t)}\right)e^{-j2\pi f_c t}\right\}.$$

Assume that the frequency response of the LPF is given by

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

By the freq.-shift property of FT,

$$x_p(t)\,e^{-j2\pi f_c t} \xrightarrow{\mathcal{F}} X_p(f-(-f_c)) = X_p(f+f_c) = \frac{1}{\sqrt{2}}\left(X_b(f+f_c-f_c) + X_b^*(-(f+f_c)-f_c)\right)$$

$$= \frac{1}{\sqrt{2}}\left(X_b(f) + X_b^*(-(f+2f_c))\right)$$

Therefore, $\sqrt{2}\,x_p(t)\,e^{-j2\pi f_c t} \xrightarrow{\mathcal{F}} X_b(f) + X_b^*(-(f+2f_c))$

$$\mathrm{LPF}\{\quad\} \xrightarrow{\mathcal{F}} X_b(f) + 0 = X_b(f)$$

Remark #1 (for Q6)

Recall the product-to-sum formula: $\cos A \cos B = \frac{1}{2}\left(\cos(A+B) + \cos(A-B)\right)$.

In this question, we will need two more formulas involving products of sine and cosine functions.

Again, we will use the Euler's identity:

$$\sin A = \frac{1}{2j}\left(e^{jA} - e^{-jA}\right)$$

$$\cos B = \frac{1}{2}\left(e^{jB} + e^{-jB}\right)$$

Hence,

$$\sin A \cos B = \frac{1}{4j}\left(e^{jA} - e^{-jA}\right)\left(e^{jB} + e^{-jB}\right) = \frac{1}{4j}\left(e^{j(A+B)} - e^{-j(A+B)} + e^{j(A-B)} - e^{-j(A-B)}\right)$$

$$= \frac{1}{4j}\left(2j\sin(A+B) + 2j\sin(A-B)\right) = \frac{1}{2}\sin(A+B) + \frac{1}{2}\sin(A-B),$$

and

$$\sin A \sin B = \frac{1}{(2j)^2}\left(e^{jA} - e^{-jA}\right)\left(e^{jB} - e^{-jB}\right) = \frac{1}{-4}\left(e^{j(A+B)} + e^{-j(A+B)} - e^{j(A-B)} - e^{-j(A-B)}\right)$$

$$= -\frac{1}{4}\left(2\cos(A+B) - 2\cos(A-B)\right) = \frac{1}{2}\left(\cos(A-B) - \cos(A+B)\right)$$

Remark #2 (for Q7)

As usual, this trig. identity can be proved via the Euler's formula:

$$\cos A \cos B = \left(\frac{e^{jA}+e^{-jA}}{2}\right)\times\left(\frac{e^{jB}+e^{-jB}}{2}\right) = \frac{1}{4}\left(e^{j(A+B)}+e^{j(A-B)}+e^{-j(A-B)}+e^{-j(A+B)}\right)$$

$$\sin A \sin B = \left(\frac{e^{jA}-e^{-jA}}{2j}\right)\times\left(\frac{e^{jB}-e^{-jB}}{2j}\right) = -\frac{1}{4}\left(e^{j(A+B)}-e^{j(A-B)}-e^{-j(A-B)}+e^{-j(A+B)}\right)$$

$$\cos A\cos B+\sin A\sin B = \frac{1}{\cancel{4}_{2}}\times\cancel{2}\left(e^{j(A-B)}+e^{-j(A-B)}\right) = \cos(A-B)$$

Remark #3 (for Q8b)

(1) The expression for $Y(f)$ above is similar to $\mathrm{Re}\{X(f)\}$ but they are not the same.

Compare:

$$\mathrm{Re}\{X(f)\} = \frac{1}{2}\left(X(f)+X^*(f)\right), \text{ and}$$

$$Y(f) = \mathcal{F}\{\mathrm{Re}\{x(t)\}\} = \frac{1}{2}\left(X(f)+X^*(\underset{\uparrow}{-}f)\right).$$

extra minus sign

(2) When $x(t)$ is real-valued,

$$y(t) = \mathrm{Re}\{x(t)\} = x(t), \text{ and}$$

$$Y(f) = \mathcal{F}\{y(t)\} = \mathcal{F}\{x(t)\} = X(f)$$

Let's check whether $Y(f)=X(f)$ if we use our derived expression for $Y(f)$.

Recall that for real-valued $x(t)$,

$$X(-f) = X^*(f)$$

So, $Y(f) = \frac{1}{2}\left(X(f)+X^*(-f)\right) = \frac{1}{2}\left(X(f)+(X^*(f))^*\right) = X(f)$. ✓

(3) Let's try another check.

Because $y(t)$ is defined as $\mathrm{Re}\{x(t)\}$, we know that $y(t)$ will always be real-valued.

Hence, it must also satisfy the conjugate symmetry property:

$$Y(-f) = Y^*(f).$$

So, let's try plugging "$-f$" into our expression for $Y(f)$:

$$Y(f) = \frac{1}{2}\left(X(f)+X^*(-f)\right)$$

This gives

$$Y(-f) = \frac{1}{2}\left(X(-f)+X^*(f)\right)$$

of course,

$$Y^*(f) = \frac{1}{2}\left(X^*(f)+X(-f)\right)$$

Therefore, $Y(-f) = Y^*(f)$ as expected.