ECS 332: Principles of Communications	**2017/1**

<div align="center">

HW 4 — Due: Not Due

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Problem 1. Consider the DSB-SC modem with no channel impairment shown in Figure 4.1. Suppose that the message is band-limited to $B = 3$ kHz and that $f_c = 100$ kHz.
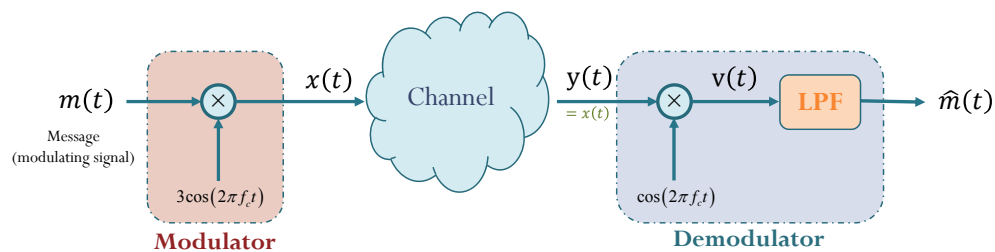

Figure 4.1: DSB-SC modem with no channel impairment

(a) Specify the frequency response $H_{LP}(f)$ of the LPF so that $\hat{m}(t) = m(t)$.

(b) Suppose the impluse response $h_{LP}(t)$ of the LPF is of the form $\alpha \operatorname{sinc}(\beta t)$. Find the constants α and β such that $\hat{m}(t) = m(t)$.

Problem 2. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 4.2. Note that V and T_b are some positive constants. Your answers should be given in terms of them.
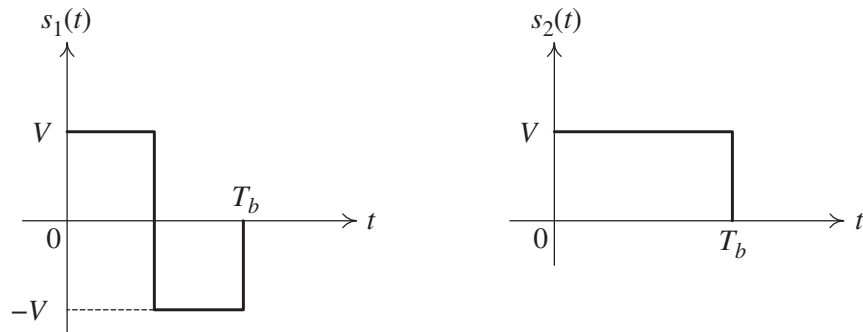
(a) Find the energy in each signal.

Figure 4.2: Signal set for Question 2

(b) Are they energy signals?

(c) Are they power signals?

(d) Find the (average) power in each signal.

(e) Are the two signals $s_1(t)$ and $s_2(t)$ orthogonal?

Problem 3. (Power Calculation) For each of the following signals $g(t)$, find (i) its corresponding power $P_g = \left\langle |g(t)|^2 \right\rangle$, (ii) the power $P_x = \left\langle |x(t)|^2 \right\rangle$ of $x(t) = g(t) \cos(10t)$, and (iii) the power $P_y = \left\langle |y(t)|^2 \right\rangle$ of $y(t) = g(t) \cos(50t)$

(a) $g(t) = 3 \cos(10t + 30°)$.

(b) $g(t) = 3\cos(10t + 30°) + 4\cos(10t + 120°)$. (Hint: First, use phasor form to combine the two components into one sinusoid.)

(c) $g(t) = 3\cos(10t) + 3\cos(10t + 120°) + 3\cos(10t + 240°)$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 4. This question starts with a ***square-modulator*** for DSB-SC . Then, the use of the square-operation block is further explored on the receiver side of the system. [Doerschuk, 2008, Cornell ECE 320]

(a) Let $x(t) = A_c m(t)$ where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Consider the block diagram shown in Figure 4.3.
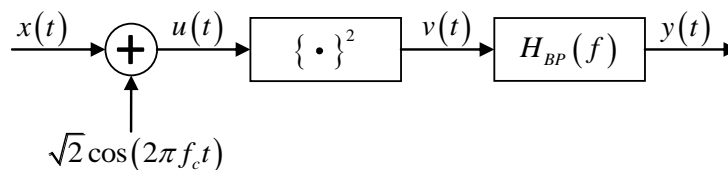

Figure 4.3: Block diagram for Problem 4a

Assume $f_c \gg B$ and

$$H_{BP}(f) = \begin{cases} 1, & |f - f_c| \leq B \\ 1, & |f + f_c| \leq B \\ 0, & \text{otherwise.} \end{cases}$$

The block labeled "$\{\cdot\}^2$" has output $v(t)$ that is the square of its input $u(t)$:

$$v(t) = u^2(t).$$

Find $y(t)$.

(b) The block diagram in part (a) provides a nice implementation of a modulator because it may be easier to build a squarer than to build a multiplier. Based on the successful use of a squaring operation in the modulator, we decide to use the same squaring operation in the demodulator. Let

$$x(t) = A_c m(t) \sqrt{2} \cos(2\pi f_c t)$$

where $m(t) \xrightleftharpoons[\mathcal{F}^{-1}]{\mathcal{F}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 4.4.
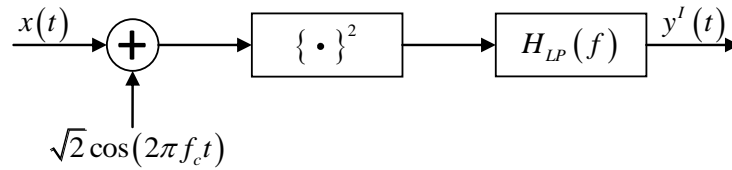
Figure 4.4: Block diagram for Problem 4b

Use

$$H_{LP}(f) = \begin{cases} 1, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^I(t)$. Does this block diagram work as a demodulator; that is, is $y^I(t)$ proportional to $m(t)$?

(c) Due to the failure in part (b), we have to think hard and it seems natural to consider also the block diagram with cos replaced by sin. Let

$$x(t) = A_c m(t) \sqrt{2} \cos(2\pi f_c t)$$

where $m(t) \xrightarrow{\mathcal{F}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$ as in part (b).
Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 4.5.

As in part (b), use

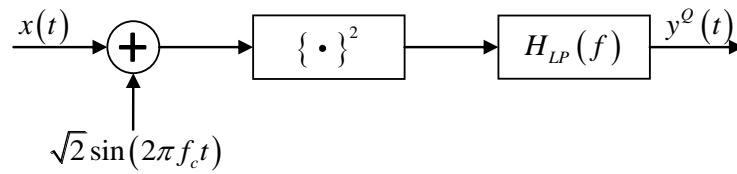$$H_{LP}(f) = \begin{cases} 1, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$$

Figure 4.5: Block diagram for Problem 4c

Find $y^Q(t)$.

(d) Use the results from parts (b) and (c). Draw a block diagram of a **_successful_** DSB-SC demodulator using squaring operations instead of multipliers.

Problem 5 (Cube modulator). Consider the block diagram shown in Figure 4.6 where "$\{\cdot\}^3$" indicates a device whose output is the cube of its input.
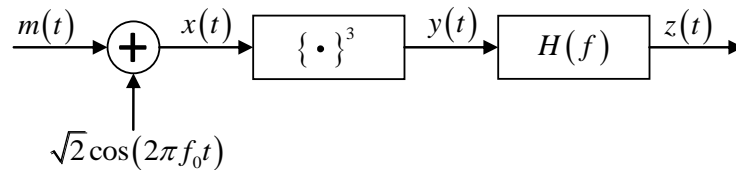

Figure 4.6: Block diagram for Problem 5. Note the use of f_0 instead of f_c.

Let $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ be bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$.

(a) Plot an $H(f)$ that gives $z(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$. What is the gain in $H(f)$? What is the value of f_c? Notice that the frequency of the cosine is f_0 not f_c. You are supposed to determine f_c in terms of f_0.

(b) Let $M(f)$ be

$$M(f) = \begin{cases} 1, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$$

(i) Plot $X(f)$.

(ii) Plot $Y(f)$. Hint:

$$M(f) * M(f) = \begin{cases} 2B - |f|, & |f| \leq 2B \\ 0, & \text{otherwise.} \end{cases}$$

Do not attempt to make an accurate plot or calculation for the Fourier transform of $m^3(t)$.

(iii) For your filter of part (a), plot $z(t)$.

[Doerschuk, 2008, Cornell ECE 320]

Problem 6. Consider a signal $g(t)$. Recall that $|G(f)|^2$ is called the **energy spectral density** of $g(t)$. Integrating the energy spectral density over all frequency gives the signal's total energy. Furthermore, the energy contained in the frequency band I can be found from the integral $\int_I |G(f)|^2 df$ where the integration is over the frequencies in band I. In particular, if the band is simply an interval of frequency from f_1 to f_2, then the energy contained in this band is given by

$$\int_{f_1}^{f_2} |G(f)|^2 df. \tag{4.1}$$

In this problem, assume

$$g(t) = 1[-1 \le t \le 1].$$

(a) Find the (total) energy of $g(t)$.

(b) Figure 4.7 define the main lobe of a sinc pulse. It is well-known that the main lobe of the sinc function contains about 90% of its total energy. Check this fact by first
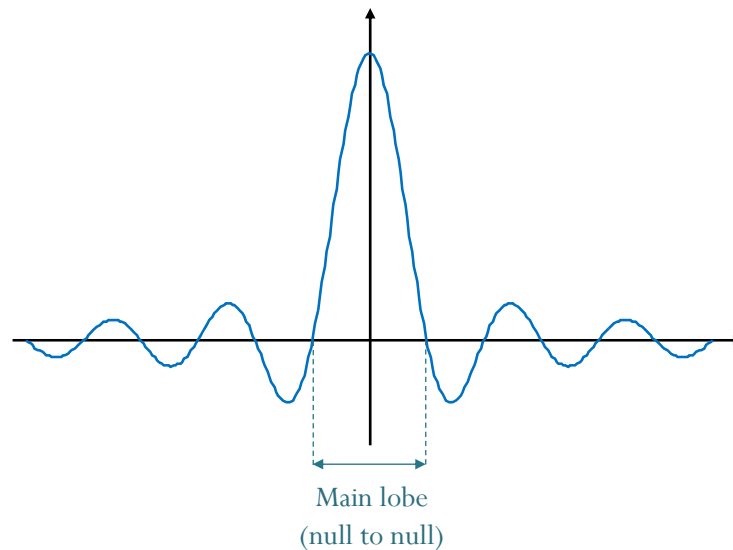
Figure 4.7: Main lobe of a sinc pulse

computing the energy contained in the frequency band occupied by the main lobe and then compare with your answer from part (a).

Hint: Find the zeros of the main lope. This give f_1 and f_2. Now, we can apply (4.1). MATLAB or similar tools can then be used to numerically evaluate the integral.

(c) Suppose we want to include more energy by considering wider frequency band. Let this band be the interval $I = [-f_0, f_0]$. Find the minimum value of f_0 that allows the band to capture at least 99% of the total energy in $g(t)$.