Q1 Nyquist's Criterion

Monday, September 17, 2012 9:47 PM

(a) In the time domain :

$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm T, \pm 2T, \pm 3T, \dots \end{cases}$$

symbol "duration" → T does not necessarily equal the pulse duration (the pulse may not even be time-limited.)

"interval"

$\frac{1}{T}$ = signaling rate measured in symbols per second or baud

(b) In the freq. domain.

$$\star \quad \sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) = T$$

Reminder : A pulse p(t) is called a Nyquist pulse iff it satisfies $\star$

Q2 Nyquist Pulses

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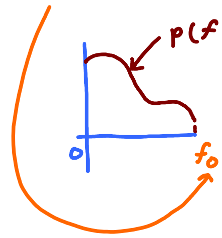
General strategy (recipe): Let the end of the given interval be $\frac{1}{2T}$.



These are chosen to ease the design. (will return to explain them later.)

Then, set the symbol duration to be $T = \frac{1}{2f_0}$

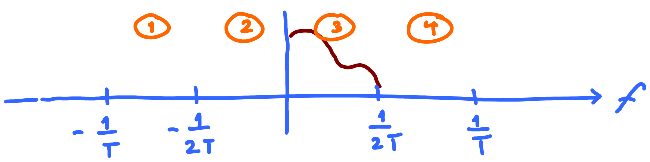
Our pulse will be band-limited to $\frac{1}{T}$.

Recall that to check whether a pulse is a Nyquist pulse,

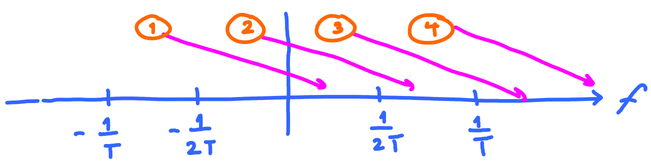
we check whether $\sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) \equiv T.$

$P(f)$ is replicated every $\frac{1}{T}$.

Now consider 4 intervals:



Note that when $P(f)$ is copied to $\frac{1}{T}$, its content in region 1 will show up in region 3



Therefore, when we add $P(f)$ in region 1 and $P(f)$ in region 3, we must have T.

In other words, $P(f)$ in region 1 can be found by $T - P(f)$

in region 3.

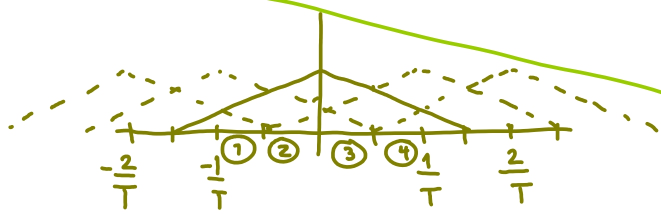
The (hinted) suggested symmetry in $P(f)$ allow us to find

$P(f)$ in region 2 by flipping $P(f)$ in region 3 horizontally

and

$P(f)$ in region 4 by flipping $P(f)$ in region 1 horizontally.

Remark. We don't want $P(f)$ to be non-zero outside region 1 - 4 because when we apply $\sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right)$, if $P(f)$ is too wide, we will have to deal with many overlapping replicas.



We don't want $P(f)$ to be zero inside region 4 (and hence 1) because we may need it to "cancel" the region 3 value to get the Σ to be T.

This is why we set the provided portion of $P(f)$ to be in region 3.

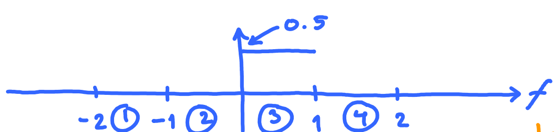
For this question, we have $f_0 = 1$. Therefore, we will choose

$T = \frac{1}{2f_0} = 0.5. \Rightarrow \frac{1}{T} = 2$ and $\frac{1}{2T} = 1.$

The four regions under consideration are
1 $f \in [-2, -1)$
2 $f \in [-1, 0)$
3 $f \in [0, 1)$
4 $f \in [1, 2]$

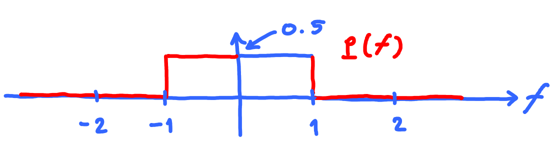
The question specifies $P(f)$ in region 3 as discussed above.
We will use the strategy described above to find $P(f)$ which is band-limited to $\frac{1}{T}$.

(a)



Note that in region 3, $P(f) \equiv T$ already. Therefore, we need nothing from region 1; in other words, $P(f) \equiv 0$ in region 1

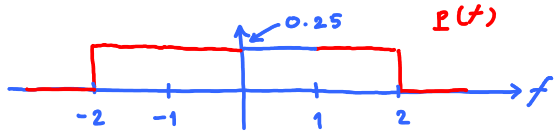
The values in region 2 and 4 are from the symmetry in $P(f)$.



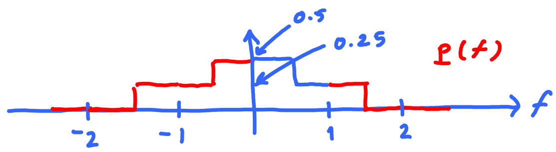
(There is no need to check whether $P(f)$ satisfies the Nyquist criterion again because it is constructed by our recipe above.)

(b) In this part, the value of $P(f)$ in region 3 is 0.25;
So, we will set the value in region 1 to be $T - 0.25 = 0.5 - 0.25 = 0.25$.

The values in region 2 and 4 are from the symmetry in $P(f)$.

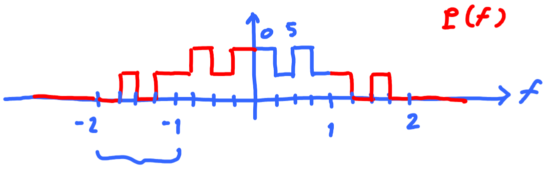


(c)



As before, the value in region 1 is T − region 3 = 0.5 −  = .

(d)



T − region 3 = 0.5 −  = 

Q3 Raised Cosine Pulse

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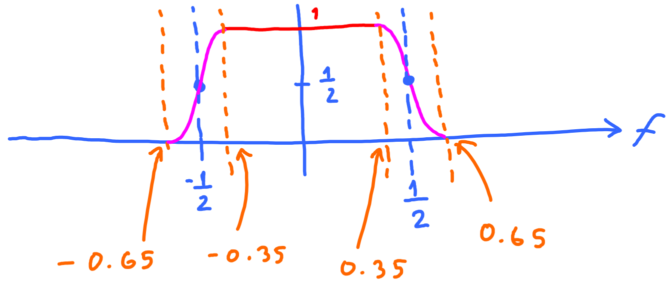
$\alpha = 0.3, \quad T = 1$

(a)

$\frac{1}{2T} = \frac{1}{2}$

$\frac{\alpha}{T} = \frac{0.3}{1} = 0.3$

$\frac{\alpha}{2T} = \frac{0.3}{2} = 0.15$



(b) The raised cosine pulse is a Nyquist pulse.

Therefore,

$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm T, \pm 2T, \pm 3T, \dots \end{cases}$$

Here, $T = 1$.

Hence,

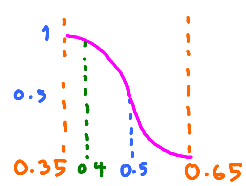
$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm 1, \pm 2, \pm 3 \dots \end{cases}$$

In particular, $p(2) = 0$

(c) Note that $P\left(\frac{1}{2T}\right) = \frac{T}{2}$. Here, $T = 1$. Hence, $P\left(\frac{1}{2}\right) = \frac{1}{2}$

(d) From part (a), we've seen that $P(f) = 1$ for $f \in [-0.35, 0.35]$. Therefore, $P(0.3) = 1$.

(e) We magnify the plot in part (a) for clarity



f: 0 π (half a cycle)

$\longleftrightarrow$

$\frac{\alpha}{T} = 0.3$

This is the "raised" part of RC.

$$\Rightarrow P(0.4) = \frac{1}{2}\left(1 + \cos\left(\pi \times \frac{0.05}{0.3}\right)\right)$$

$$= \frac{1}{2}\left(1 + \cos\left(\pi/6\right)\right)$$

$$= \frac{2+\sqrt{3}}{4} \approx 0.933$$

Alternatively, although not recommended, we can find the value of $P(0.4)$ simply by plugging in the formula for the raised cosine given in lecture.

The value of P in the cosine part is given by $\frac{T}{2}\left(1+\cos\left(\frac{\pi T}{\alpha}\left(|f| - \frac{1-\alpha}{2T}\right)\right)\right)$.

In this part, $f = 0.4$, $\alpha = 0.3$, and $T = 1$.

Hence,

$$P(0.4) = \frac{1}{2}\left(1+\cos\left(\frac{\pi\times 1}{0.3}\underbrace{\left(0.4 - \frac{0.7}{2}\right)}_{0.05}\right)\right) = \frac{1}{2}\left(1+\cos\left(\frac{\pi}{6}\right)\right) \leftarrow \text{same as above.}$$

Note: $\cos\left(\frac{\pi}{6}\right) \approx 0.866$

this is in radians (not degrees)

If you plug $\frac{\pi}{6}$ into the cos function when your calculator is in deg mode, you would get the incorrect number $\cos\left(\frac{\pi}{6}^{\circ}\right) \approx 1$

(0.999958244)

Q4 Raised Cosine Pulse

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(a) The formula for the raised cosine pulse in the time domain is

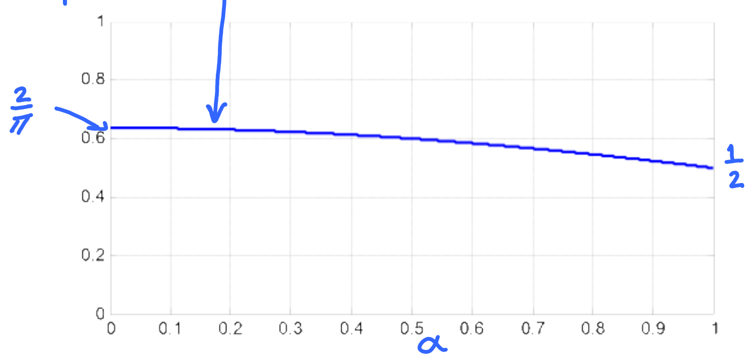
$$p_{RC}(t;\alpha) = \frac{\cos\left(\alpha\pi\frac{t}{T}\right)}{1-\left(2\alpha\frac{t}{T}\right)^2}\,\frac{\sin\left(\pi\frac{t}{T}\right)}{\pi\frac{t}{T}}.$$

At $t=\frac{T}{2}$, we have $\frac{t}{T}=\frac{1}{2}$

$$p_{RC}\left(\tfrac{T}{2};\alpha\right) = \frac{\cos\left(\alpha\frac{\pi}{2}\right)}{1-\alpha^2}\,\frac{\overbrace{\sin\frac{\pi}{2}}^{1}}{\frac{\pi}{2}}$$

$$= \frac{2}{\pi}\times\frac{\cos\left(\alpha\frac{\pi}{2}\right)}{1-\alpha^2}$$

(b) MATLAB plot:



(c) Note that when $\alpha=0$, $p_{RC}\left(\frac{T}{2};\alpha\right)=\frac{2}{\pi}\approx 0.6366$

As $\alpha\to 1$, $p_{RC}\left(\frac{T}{2};\alpha\right) \to \cancel{\frac{2}{\pi}}\times\frac{\cancel{\frac{\pi}{2}}\left(-\sin\left(\alpha\frac{\pi}{2}\right)\right)}{-2\alpha}\Bigg|_{\alpha=1} = \frac{1}{2}.$