ECS 332: Principles of Communications
HW 9 - Due: December 2 (Friday), 5PM
Lecturer: Prapun Suksompong, Ph.D.

## Instructions

(a) This assignment has 4 pages.
(b) (1 pt) Write your first name and the last three digit of your student ID on the upperright corner of every submitted sheet.
(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.
(d) $(8 \mathrm{pt})$ It is important that you try to solve all non-optional problems.
(e) Late submission will be heavily penalized.

Problem 1. State the Nyquist's (first) criterion for zero ISI
(a) In the time domain.
(b) In the frequency domain.

Problem 2. In each part below, a pulse $P(f)$ is defined in the frequency domain from $f=0$ to $f=1$. Outside of $[0,1]$, your task is to assign value(s) to $P(f)$ so that it becomes a Nyquist pulse. Of course, you will also need to specify the symbol interval $T$ as well.

Hint: To avoid dealing with complex-valued $P(f)$, you may assume that $p(t)$ is real-valued and even; in which case $P(f)$ is also real-valued and even.
(a) Find a Nyquist pulse $P(f)$ whose $P(f)=0.5$ on $[0,1]$.
(b) Find a Nyquist pulse $P(f)$ whose $P(f)=0.25$ on $[0,1]$.
(c) Find a Nyquist pulse $P(f)$ whose

$$
P(f)= \begin{cases}0.5, & 0 \leq f<0.5 \\ 0.25, & 0.5 \leq f \leq 1\end{cases}
$$

(d) Find a Nyquist pulse $P(f)$ whose

$$
P(f)= \begin{cases}0.5, & f \in[0,0.25) \cup[0.5,0.75) \\ 0.25, & f \in[0.25,0.5) \cup[0.75,1]\end{cases}
$$

Problem 3. Consider a raised cosine pulse $p_{\mathrm{RC}}(t ; \alpha)$ and its Fourier transform $P_{\mathrm{RC}}(f ; \alpha)$. Assume the rolloff factor $\alpha=0.3$ and the symbol "duration" $T=1$.
(a) Carefully sketch $P_{\mathrm{RC}}(f ; \alpha)$.
(b) Find $p_{\mathrm{RC}}(2 ; \alpha)$.
(c) Find $P_{\mathrm{RC}}(0.5 ; \alpha)$.
(d) Find $P_{\mathrm{RC}}(0.3 ; \alpha)$.
(e) *Find $P_{\mathrm{RC}}(0.4 ; \alpha)$.

Remark: You should be able to solve this problem without referring to the "ugly" expression (9.1) below.

## Extra Question

Here is an optional question for those who want more practice.
Problem 4. Consider a raised cosine pulse $p_{\mathrm{RC}}(t ; \alpha)$ with rolloff factor $\alpha$ and symbol "duration" $T$. Its time domain expression is

$$
\begin{equation*}
p_{\mathrm{RC}}(t ; \alpha)=\frac{\cos \frac{\alpha \pi t}{T}}{1-\frac{4 \alpha^{2} t^{2}}{T^{2}}} \operatorname{sinc} \frac{\pi t}{T}=\frac{\cos \frac{\alpha \pi t}{T}}{1-\frac{4 \alpha^{2} t^{2}}{T^{2}}} \frac{\sin \frac{\pi t}{T}}{\frac{\pi t}{T}} . \tag{9.1}
\end{equation*}
$$

(a) Find $p(T / 2)$ as a function of $\alpha$.
(b) Use MATLAB to plot $p(T / 2)$ as a function of $\alpha$.
(c) Find $\lim _{\alpha \rightarrow 1} p_{\mathrm{RC}}\left(\frac{T}{2} ; \alpha\right)$.

