ECS 332: Principles of Communications **2016/1**

HW 6 — Due: November 4, 5PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 6 pages.

(b) (1 pt) Write your first name and the last three digit of your student ID on the upper-right corner of ***every*** submitted page.

(c) (1 pt) For each part, write your explanation/derivation and answer in the space provided.

(d) (8 pt) It is important that you try to solve all problems.

(e) Late submission will be heavily penalized.

Problem 1. You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos(2\pi f_c t)$, where $m(t)$ is a signal band-limited to B Hz. Figure 6.1 shows a DSB-SC modulator available in the stockroom. Note that, as usual, $\omega_c = 2\pi f_c$. The carrier generator available generates not $\cos(2\pi f_c t)$, but $\cos^3(2\pi f_c t)$. Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like. [Lathi and Ding, 2009, Q4.2-3]

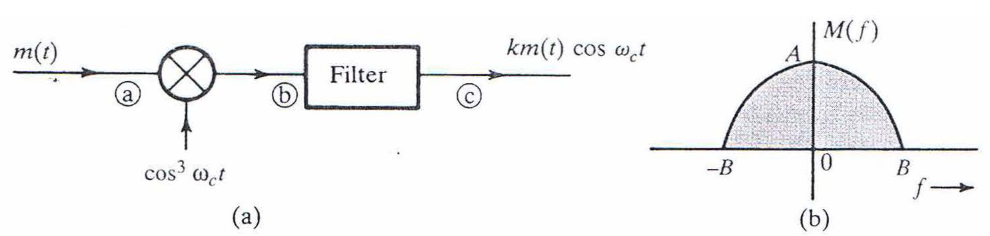

Figure 6.1: Problem 1

(a) What kind of filter is required in Figure 6.1?

(b) Determine the signal spectra at points (b) and (c), and indicate the frequency bands occupied by these spectra.

(c) What is the minimum usable value of f_c?

(d) Would this scheme work if the carrier generator output were $\cos^2 \omega_c t$? Explain.

Problem 2. Consider an AM transmitter.

(a) Suppose the message $m(t) = 4\cos(10\pi t)$ and the transmitted signal is

$$x_{\mathrm{AM}}(t) = A\cos(100\pi t) + m(t)\cos(100\pi t).$$

Find the value of A which yields the modulation index in each part below.

(i) $\mu = 50\%$

(ii) $\mu = 100\%$

(iii) $\mu = 150\%$

(b) Suppose the message $m(t) = \alpha\cos(10\pi t)$ and the transmitted signal is

$$x_{\mathrm{AM}}(t) = 4\cos(100\pi t) + m(t)\cos(100\pi t).$$

Find the value of α which yields the modulation index in each part below.

(i) $\mu = 50\%$

(ii) $\mu = 100\%$

(iii) $\mu = 150\%$

Problem 3. Consider the basic DSB-SC transceiver with time-delay channel presented in class. Recall that the input of the receiver is

$$x(t-\tau) = m(t-\tau)\sqrt{2}\cos(\omega_c(t-\tau))$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

(a) Suppose that, at the receiver, we multiply by $\sqrt{2}\cos((\omega_c t) - \theta)$ instead of $\sqrt{2}\cos(\omega_c t)$ as illustrated in Figure 6.2. Assume

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $\hat{m}(t)$ (the output of the LPF).

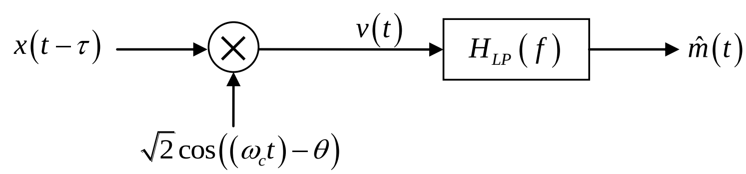

Figure 6.2: Receiver for Problem 3a

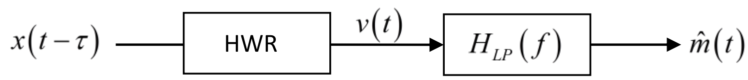

Figure 6.3: Receiver for Problem 3b

(b) Use the same assumptions as part (a). However, at the receiver, instead of multiplying by $\sqrt{2}\cos((\omega_c t) - \theta)$, we pass $x(t-\tau)$ through a half-wave rectifier (HWR) as shown in Figure 6.3.

Make an extra assumption that $m(t) \geq 0$ for all time t and that the half-wave rectifier input-output relation is described by a function $f(\cdot)$:

$$f(x) = \begin{cases} x, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Find $\hat{m}(t)$ (the output of the LPF).

Extra Questions

Here are some optional questions for those who want more practice.

Problem 4. Would the scheme in Problem 1 work if the carrier generator output were $\cos^n \omega_c t$ for any integer $n \geq 2$?

Problem 5. Solve Problem 7 in [M2011].