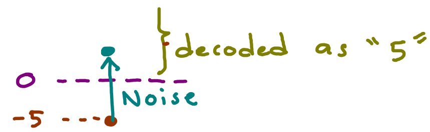
Q1 Digital PAM with Noise

Wednesday, December 02, 2015 1:27 PM



(a) The decoded value is "5". Therefore, the received signal must be > 0 ← threshold

Because the transmitted value was "-5",

the additive noise must be > 5 to give received signal > 0.

$$P[N > 5] = Q\left(\frac{5}{\sigma_N}\right) = Q\left(\frac{5}{3}\right) = Q(1.67) \approx 0.0475$$

(b) The decoded value is "5". Therefore, the received signal must be > 0

Because the transmitted value was "5",

the additive noise must be > -5 to give received signal > 0.

$$P[N > -5] = Q\left(-\frac{5}{\sigma_N}\right) = 1 - Q\left(\frac{5}{3}\right) \approx 0.9525$$

We don't like answer with negative argument in the Q function.
So, we use the property $Q(-z) = 1 - Q(z)$

(c) The decoded value is "-5". Therefore, the received signal must be < 0

Because the transmitted value was "-5",

the additive noise must be < 5 to give received signal < 0.

$$P[N < 5] = 1 - P[N \geq 5] = 1 - Q\left(\frac{5}{\sigma_N}\right) = 1 - Q\left(\frac{5}{3}\right) \approx 0.9525$$

Don't need equality because N is a cont. RV. $P[N = 5] = 0$.

(d) The decoded value is "-5". Therefore, the received signal must be < 0

Because the transmitted value was "5",

the additive noise must be < -5 to give received signal < 0.

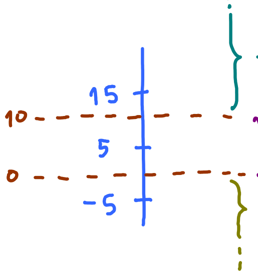
$$P[N < -5] = 1 - P[N \geq -5] = 1 - Q\left(\frac{-5}{\sigma_N}\right) = 1 - \left(1 - Q\left(\frac{5}{\sigma_N}\right)\right) = Q\left(\frac{5}{\sigma_N}\right)$$

$$= Q\left(\frac{5}{3}\right) \approx 0.0475$$

(e) $P(\mathcal{E}) = Q\left(\frac{a}{\sigma_N}\right) = Q\left(\frac{5}{3}\right) \approx 0.0475$

(f) Recall that we got the treshold level from the "minimum-distance-decoding" principle. Because it is more likely that the noise value will be small, the most likely transmitted value is the one that is closest to the received value.

Here, there are three possibilities: $-5, 5, 15$



← If the (sampled) received value is >10, the closest value in the alphabet is 15.

← If the (sampled) received value is between 0 and 10, the closest value in the alphabet is 5.

← If the (sampled) received value is <0, the closest value in the alphabet is -5.

Therefore, we should have two thresholds: 0 and 10.

$$\hat{m}[n] = \begin{cases} 15, & y[n] > 10, \\ 5, & 0 \leq y[n] \leq 10, \\ -5, & y[n] < 0. \end{cases}$$

Note that the decoding when $y[n]$ is right at the threshold levels ($y[n]=0$ and $y[n]=10$) is not unique. For example, when $y[n]=0$, it has the same probability of coming from $m[n]=-5$ or $m[n]=5$. (We will revisit this in more details in ECS 452.)

In the above expression, we choose to decode $y[n]=0$ as $\hat{m}[n]=5$. Similarly, we choose to decode $y[n]=10$ as $\hat{m}[n]=5$ as well.

Q2 SER and BER

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(a) There are 14 symbols in this string.

Don't forget to count "space" and "."

Two errors occur : $o \rightarrow i$

$v \rightarrow k$

So, the symbol error rate is $\frac{2}{14} = \frac{1}{7}$

(b) Each symbol corresponds to 7 bits. So, we have 14×7 bits.

Two symbol errors occur : o (1101111) $\longrightarrow$ i (1101001)

v (1110110) $\longrightarrow$ k (1101011)

Counting ~~#~~ bit errors, we have 6 bit errors.

So, the bit error rate is $\frac{\cancel{6}^{3}}{\cancel{14}_{7} \times 7} = \frac{3}{49}$