ECS 332: Principles of Communications   **2015/1**

HW 7 — Due: November 19, 12:59 PM (right before class)

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)
    The extra question at the end is optional.

(c) Late submission will not be accepted.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

(e) For questions that require the use of `MATLAB`,

    (i) the codes used should be printed,

    (ii) if the code is based on some provided code, all modified lines should be highlighted or underlined,

    (iii) the results displayed in the command window and the figures generated should be captured and printed out as well.

Problem 1. Recall that, in QAM system, the transmitted signal is of the form

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

In class, we have shown that

$$\text{LPF}\left\{x_{\text{QAM}}(t)\sqrt{2}\cos(2\pi f_c t)\right\} = m_1(t).$$

Give a similar proof to show that

$$\text{LPF}\left\{x_{\text{QAM}}(t)\sqrt{2}\sin(2\pi f_c t)\right\} = m_2(t).$$

Problem 2. Recall that, in QAM system, the transmitted signal is of the form

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

We want to express x_{QAM} in the form

$$x_{\text{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \phi(t)),$$

where $E(t) \geq 0$ and $\phi(t) \in (-180°, 180°]$. (This shows that QAM can be expressed as a combination of amplitude modulation and phase modulation.)

In each part below, we consider different examples of the messages $m_1(t)$ and $m_2(t)$.

(a) Consider $m_1(t)$ and $m_2(t)$ plotted in Figure 7.1. Draw the corresponding $E(t)$ and $\phi(t)$.

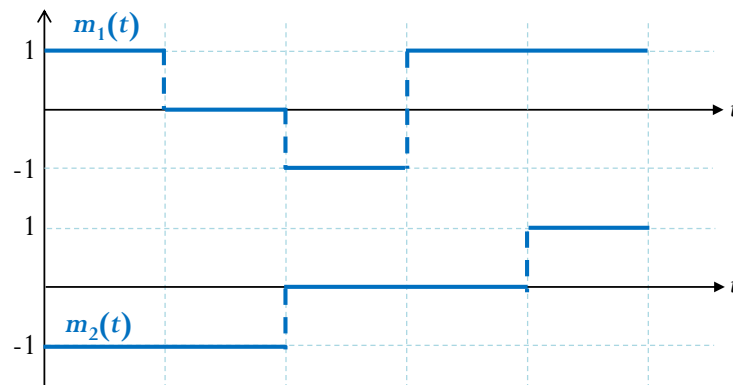Figure 7.1: $m_1(t)$ and $m_2(t)$ for part (a) of Problem 2

(b) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = \sin(2\pi Bt)$ where $0 < B \ll f_c$. Find $E(t)$ and $\phi(t)$.

Hint: $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha - \beta)$

(c) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = 2\sin(2\pi Bt)$. Let $f_c = 5$ and B $= 2$. Use MATLAB to plot the corresponding $E(t)$ and $\phi(t)$ from $t = 0$ to $t = 5$ [sec]. (Hint: the function `angle` or `atan2` will be helpful here.)

Problem 3. In **quadrature amplitude modulation** (**QAM**) or **quadrature multiplexing**, two baseband signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the following QAM signal:

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t).$$

An error in the phase or the frequency of the carrier at the demodulator in QAM will result in loss and interference between the two channels (cochannel interference).

In this problem, show that

$$\text{LPF}\left\{x_{\text{QAM}}(t)\sqrt{2}\cos((\omega_c + \Delta\omega)t + \delta)\right\} = m_1(t)\cos((\Delta\omega)t + \delta) - m_2(t)\sin((\Delta\omega)t + \delta)$$

$$\text{LPF}\left\{x_{\text{QAM}}(t)\sqrt{2}\sin((\omega_c + \Delta\omega)t + \delta)\right\} = m_1(t)\sin((\Delta\omega)t + \delta) + m_2(t)\cos((\Delta\omega)t + \delta).$$

Problem 4. Consider the message $m(t)$ along with the carrier signal $\cos(2\pi f_c t + \phi)$ plotted in Figure 7.2.
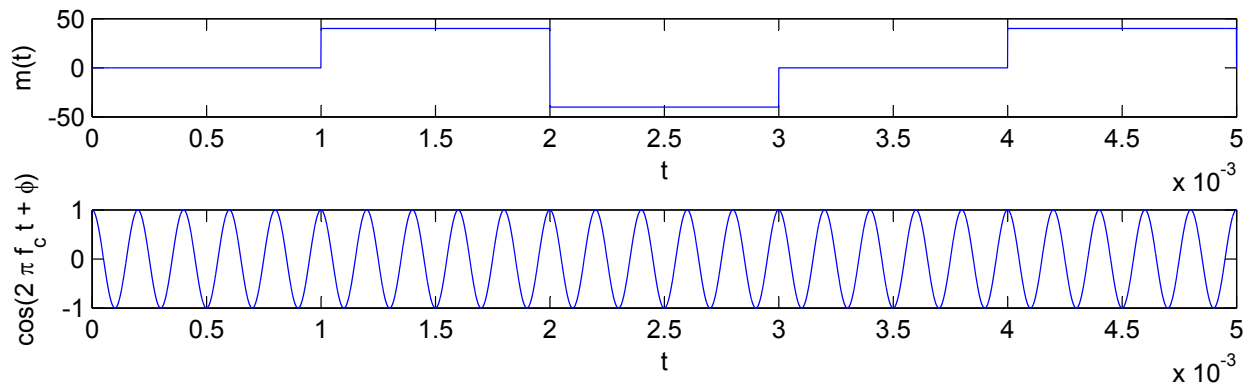

Figure 7.2: The message and the carrier signals for Problem 4.

(a) Find the carrier frequency f_c from the plot. (Hint: It is an integer.)

(b) Sketch the following signals. Make sure that (the unspecified parameter(s) are selected such that) the important "features" of the graphs can be seen clearly.

(i) $x_{\text{AM}}(t) = (A + m(t))\cos(2\pi f_c t + \phi)$ whose modulation index $\mu = 100\%$.

(ii) $x_{\text{FM}}(t) = A\cos\left(2\pi f_c t + \phi + 2\pi k_f \int\limits_{-\infty}^{t} m(\tau)d\tau\right)$.

- You may assume $m(t) = 0$ for $t < 0$.

(iii) $x_{\text{PM}}(t) = A\cos(2\pi f_c t + \phi + k_p m(t))$ with $k_p = \frac{\pi}{m_p}$.

(c) Check your answers in part (b) using MATLAB.
(Hint: The m-file for the quiz solution can be used as a starting point. There, the integration is done numerically.)

- For $x_{\text{FM}}(t)$, use $k_f = 200$.

Problem 5. Consider the FM transmitted signal

$$x_{\text{FM}}(t) = A\cos\left(2\pi f_c t + \phi + 2\pi k_f \int\limits_{-\infty}^{t} m(\tau)d\tau\right),$$

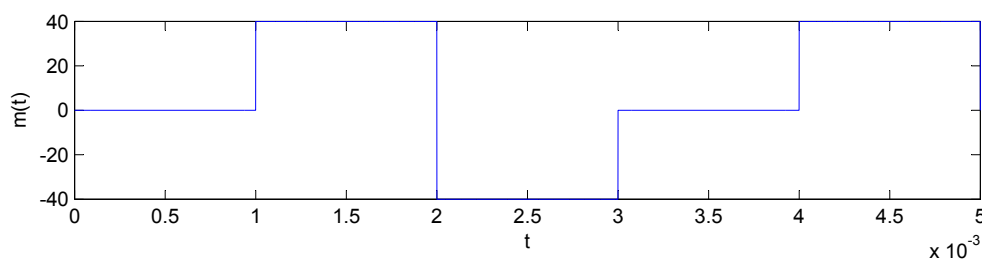where $f_c = 5$ [kHz], $A = 1$, and $k_f = 75$. The message $m(t)$ is shown in Figure 7.3.

Figure 7.3: The message $m(t)$ for Problem 5

The magnitude spectrum $|X_{\text{FM}}(f)|$ is plotted in Figure 7.4.

(a) Find the values of f_1, f_2, and f_3.
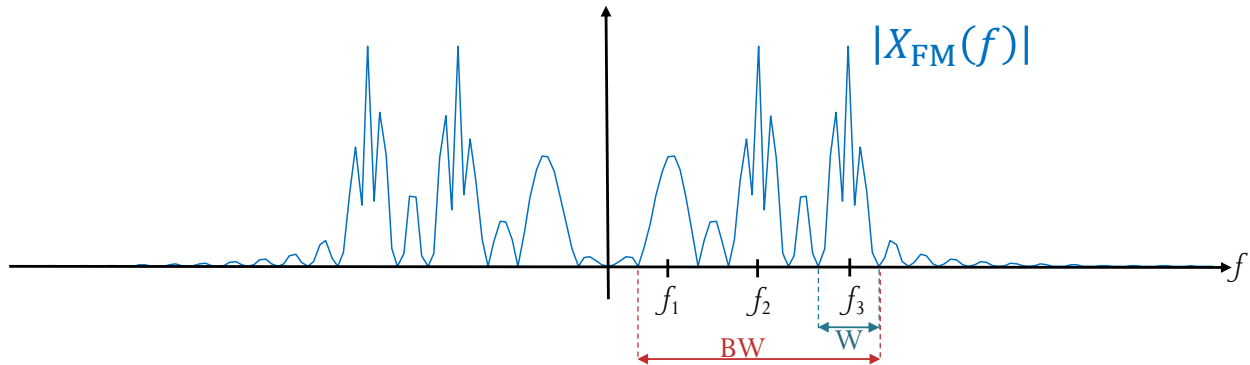
(b) Find the width W in Figure 7.4.

Figure 7.4: The magnitude spectrum $|X_{\text{FM}}(f)|$ for Problem 5

(c) Find the occupied bandwidth denoted by BW in Figure 7.4.

Extra Questions

Here are some optional questions for those who want more practice.

Problem 6. Consider a complex-valued signal $x(t)$ whose Fourier transform is $X(f)$.

(a) Find and simplify the Fourier transform of $x^*(t)$.

(b) Find and simplify the Fourier transform of $\text{Re}\{x(t)\}$.

   - Hint: $x(t) + x^*(t) = ?$

Problem 7. Consider a (complex-valued) baseband signal $x_b(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} X_b(f)$ which is band-limited to B, i.e., $|X_b(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

(a) The passband signal $x_p(t)$ is given by

$$x_p(t) = \sqrt{2}\,\text{Re}\left\{e^{j2\pi f_c t} x_b(t)\right\}.$$

   Find and simplify the Fourier transform of $x_p(t)$.

(b) Find and simplify

$$\text{LPF}\left\{\sqrt{2}\left(\underbrace{\sqrt{2}\,\text{Re}\left\{e^{j2\pi f_c t} x_b(t)\right\}}_{x_p(t)}\right)e^{-j2\pi f_c t}\right\}.$$

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Assume that the frequency response of the LPF is given by

$$H_{LP}(f) = \begin{cases} 1, & |f| \leq B \\ 0, & \text{otherwise.} \end{cases}$$