ECS 332: Principles of Communications **2015/1**

HW 6 — Due: November 3, 10:39 AM (in class)

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)
The extra question at the end is optional.

(c) Late submission will not be accepted.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Consider a "square" wave (a train of rectangular pulses) shown in Figure 6.1. Its values periodically alternates between two values A and 0. At $t = 0$, its value is A.

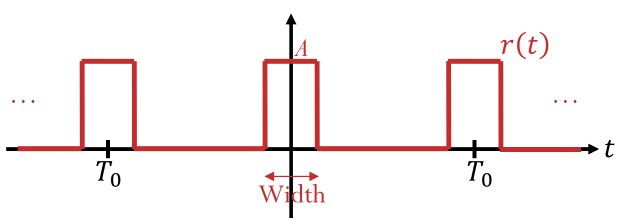

Figure 6.1: A train of rectangular pulses

Some values of its Fourier series coefficients are provided in the table below:

k	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
c_k	$-\frac{\sqrt{2}}{7\pi}$	$-\frac{1}{3\pi}$	$-\frac{\sqrt{2}}{5\pi}$	0	$\frac{\sqrt{2}}{3\pi}$	$\frac{1}{\pi}$	$\frac{\sqrt{2}}{\pi}$	$\frac{1}{2}$	$\frac{\sqrt{2}}{\pi}$	$\frac{1}{\pi}$	$\frac{\sqrt{2}}{3\pi}$	0	$-\frac{\sqrt{2}}{5\pi}$	$-\frac{1}{3\pi}$	$-\frac{\sqrt{2}}{7\pi}$

(a) Find its duty cycle.

(b) Find the value of A. (Hint: Use c_0.)

Problem 2. You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos(2\pi f_c t)$, where $m(t)$ is a signal band-limited to B Hz. Figure 6.2 shows a DSB-SC modulator available in the stockroom. Note that, as usual, $\omega_c = 2\pi f_c$. The carrier generator available generates not $\cos(2\pi f_c t)$, but $\cos^3(2\pi f_c t)$. Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like.

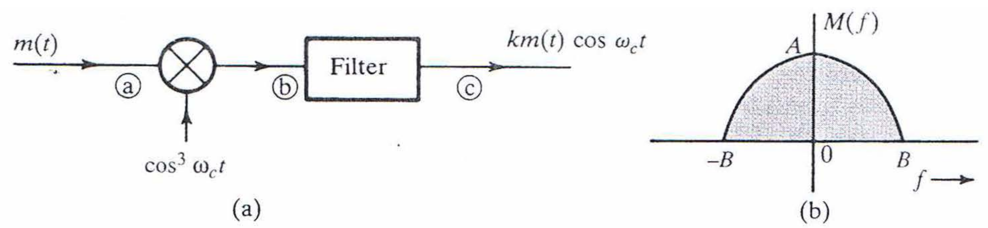

Figure 6.2: Problem 2

(a) What kind of filter is required in Figure 6.2?

(b) Determine the signal spectra at points (b) and (c), and indicate the frequency bands occupied by these spectra.

(c) What is the minimum usable value of f_c?

(d) Would this scheme work if the carrier generator output were $\cos^2 \omega_c t$? Explain.

(e) (Optional) Would this scheme work if the carrier generator output were $\cos^n \omega_c t$ for any integer $n \geq 2$?

[Lathi and Ding, 2009, Q4.2-3]

Problem 3. Consider the basic DSB-SC transceiver with time-delay channel presented in class. Recall that the input of the receiver is

$$x(t-\tau) = m(t-\tau)\sqrt{2}\cos(\omega_c(t-\tau))$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

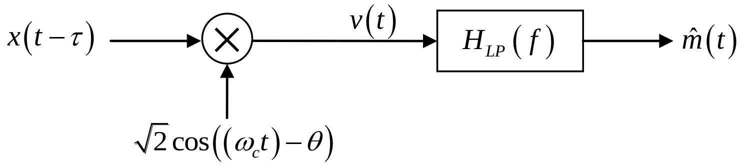

Figure 6.3: Receiver for Problem 3a

(a) Suppose that, at the receiver, we multiply by $\sqrt{2}\cos((\omega_c t) - \theta)$ instead of $\sqrt{2}\cos(\omega_c t)$ as illustrated in Figure 6.3. Assume

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $\hat{m}(t)$ (the output of the LPF).

(b) Use the same assumptions as part (a). However, at the receiver, instead of multiplying by $\sqrt{2}\cos((\omega_c t) - \theta)$, we pass $x(t-\tau)$ through a half-wave rectifier (HWR) as shown in Figure 6.4.

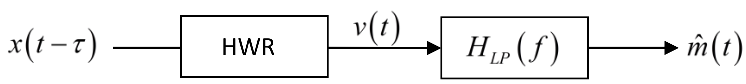

Figure 6.4: Receiver for Problem 3b

Make an extra assumption that $m(t) \ge 0$ for all time t and that the half-wave rectifier input-output relation is described by a function $f(\cdot)$:

$$f(x) = \begin{cases} x, & x \ge 0, \\ 0, & x < 0. \end{cases}$$

Find $\hat{m}(t)$ (the output of the LPF).

Problem 4. Solve Question 7 in [M2011].

Problem 5. Consider an AM transmitter.

(a) Suppose the message $m(t) = 4\cos(10\pi t)$ and the transmitted signal is

$$x_{\text{AM}}(t) = A\cos(100\pi t) + m(t)\cos(100\pi t).$$

Find the value of A which yields the modulation index in each part below.

(i) $\mu = 50\%$

(ii) $\mu = 100\%$

(iii) $\mu = 150\%$

(b) Suppose the message $m(t) = \alpha\cos(10\pi t)$ and the transmitted signal is

$$x_{\text{AM}}(t) = 4\cos(100\pi t) + m(t)\cos(100\pi t).$$

Find the value of α which yields the modulation index in each part below.

(i) $\mu = 50\%$

(ii) $\mu = 100\%$

(iii) $\mu = 150\%$

Extra Question

Here is an optional question for those who want more practice.

Problem 6 (M2011)**.** In this question, you are provided with a partial proof of an important result in the study of Fourier transform. Your task is to figure out the quantities/expressions inside the boxes labeled a,b,c, and d.

We start with a function $g(t)$. Then, we define $x(t) = \sum\limits_{\ell=-\infty}^{\infty} g(t - \ell T)$. It is a sum that involves $g(t)$. What you will see next is our attempt to find another expression for $x(t)$ in terms of a sum that involves $G(f)$.

To do this, we first write $x(t)$ as $x(t) = g(t) * \sum\limits_{\ell=-\infty}^{\infty} \delta(t - \ell T)$. Then, by the convolution-in-time property, we know that $X(f)$ is given by

$$X(f) = G(f) \times \boxed{\quad a \quad} \sum_{\ell=-\infty}^{\infty} \delta\left(f + \boxed{\qquad b \qquad}\right)$$

We can get $x(t)$ back from $X(f)$ by the inverse Fourier transform formula: $x(t) = \int\limits_{-\infty}^{\infty} X(f)\, e^{j2\pi ft} df$. After plugging in the expression for $X(f)$ from above, we get

$$\begin{aligned} x(t) &= \int_{-\infty}^{\infty} e^{j2\pi ft} G(f) \boxed{\quad a \quad} \sum_{\ell=-\infty}^{\infty} \delta\left(f + \boxed{\qquad b \qquad}\right) df \\ &= \boxed{\quad a \quad} \int_{-\infty}^{\infty} \sum_{\ell=-\infty}^{\infty} e^{j2\pi ft} G(f)\, \delta\left(f + \boxed{\qquad b \qquad}\right) df. \end{aligned}$$

By interchanging the order of summation and integration, we have

$$x(t) = \boxed{\quad a \quad} \sum_{\ell=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j2\pi ft} G(f)\, \delta\left(f + \boxed{\qquad b \qquad}\right) df.$$

We can now evaluate the integral via the sifting property of the delta function and get

$$x(t) = \boxed{\quad a \quad} \sum_{\ell=-\infty}^{\infty} e^{\boxed{\quad c \quad}} G\left(\boxed{\quad d \quad}\right).$$