

HW 5 — Due: Not Due

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Problem 1. Consider the two signals $s_1(t)$ and $s_2(t)$ shown in Figure 5.1. Note that V and T_b are some positive constants. Your answers should be given in terms of them.

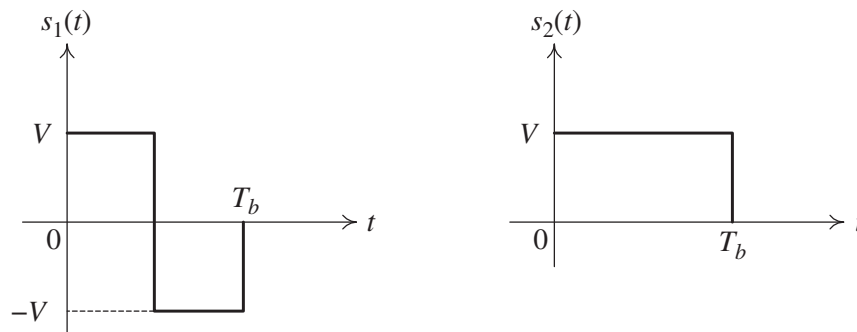


Figure 5.1: Signal set for Question 1

- Find the energy in each signal.
- Are they energy signals?
- Are they power signals?
- Find the (average) power in each signal.
- Are the two signals $s_1(t)$ and $s_2(t)$ orthogonal?

Problem 2. (Power Calculation) Find the average power of each of the signals given below.

- $g(t) = 2e^{2jt}$
- $g(t) = 2 \cos(2t + 2^\circ)$
- $g(t) = 2 \cos(2t + 2^\circ) + 2 \cos(2t + 2^\circ)$
- $g(t) = 2 \cos(2t + 2^\circ) + 22 \cos(22t + 22^\circ)$

Problem 3. (Power Calculation) For each of the following signals $g(t)$, find (i) its corresponding power $P_g = \langle |g(t)|^2 \rangle$, (ii) the power $P_x = \langle |x(t)|^2 \rangle$ of $x(t) = g(t) \cos(10t)$, and (iii) the power $P_y = \langle |y(t)|^2 \rangle$ of $y(t) = g(t) \cos(50t)$

(a) $g(t) = 3 \cos(10t + 30^\circ)$.

(b) $g(t) = 3 \cos(10t + 30^\circ) + 4 \cos(10t + 120^\circ)$. (Hint: First, use phasor form to combine the two components into one sinusoid.)

(c) $g(t) = 3 \cos(10t) + 3 \cos(10t + 120^\circ) + 3 \cos(10t + 240^\circ) = 0$

Problem 4. Consider a signal $g(t)$. Recall that $|G(f)|^2$ is called the **energy spectral density** of $g(t)$. Integrating the energy spectral density over all frequency gives the signal's total energy. Furthermore, the energy contained in the frequency band I can be found from the integral $\int_I |G(f)|^2 df$ where the integration is over the frequencies in band I . In particular, if the band is simply an interval of frequency from f_1 to f_2 , then the energy contained in this band is given by

$$\int_{f_1}^{f_2} |G(f)|^2 df. \quad (5.1)$$

In this problem, assume

$$g(t) = 1[-1 \leq t \leq 1].$$

(a) Find the (total) energy of $g(t)$.

(b) Figure 5.2 define the main lobe of a sinc pulse. It is well-known that the main lobe of the sinc function contains about 90% of its total energy. Check this fact by first computing the energy contained in the frequency band occupied by the main lobe and then compare with your answer from part (a).

Hint: Find the zeros of the main lobe. This give f_1 and f_2 . Now, we can apply (5.1). MATLAB or similar tools can then be used to numerically evaluate the integral.

(c) Suppose we want to include more energy by considering wider frequency band. Let this band be the interval $I = [-f_0, f_0]$. Find the minimum value of f_0 that allows the band to capture at least 99% of the total energy in $g(t)$.

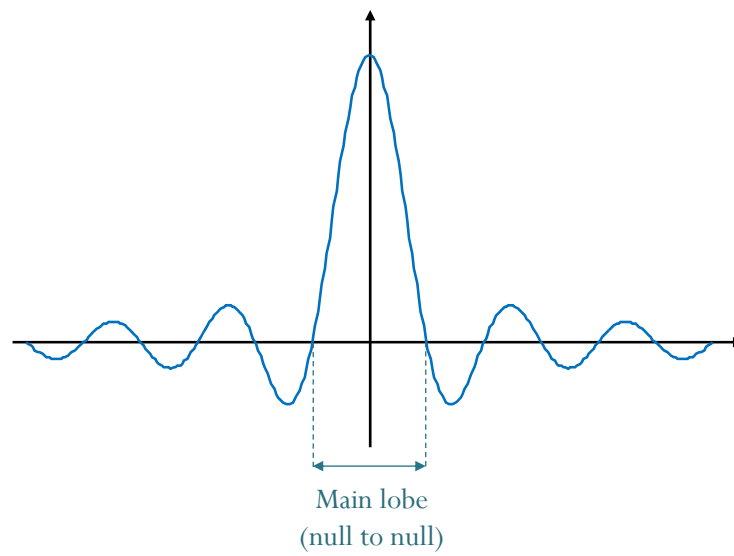


Figure 5.2: Main lobe of a sinc pulse