ECS 332: Principles of Communications	**2015/1**

HW 4 — Due: September 22, 10:39 AM (in class)

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)
The extra questions at the end are optional.

(c) Late submission will not be accepted.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. Given a system with input-output relationship of

$$y(t) = 2x(t) + 10,$$

is this system linear? [Carlson and Crilly, 2009, Q2.3-10]

Problem 2. Signal $x(t) = 10\cos(2\pi \times 7 \times 10^6 \times t)$ is transmitted to some destination. The received signal is $y(t) = 10\cos(2\pi \times 7 \times 10^6 \times t - \pi/6)$.

(a) What is the minimum distance between the source and destination?

(b) What are the other possible distances?

[Carlson and Crilly, 2009, Q2.3-14]

Problem 3. Consider the DSB-SC modem with no channel impairment shown in Figure 4.1. Suppose that the message is band-limited to $B = 3$ kHz and that $f_c = 100$ kHz.

(a) Specify the frequency response $H_{LP}(f)$ of the LPF so that $\hat{m}(t) = m(t)$.

(b) Suppose the transfer function $h_{LP}(t)$ of the LPF is of the form $\cancel{A}\,\alpha\operatorname{sinc}(\cancel{B}\,\beta t)$. Find the constants $\cancel{A}\,\alpha$ and $\cancel{B}\,\beta$ such that $\hat{m}(t) = m(t)$.

The B here is a different constant from the B = 3 kHz that defines M(f). So, we change the name to β to prevent confusion.

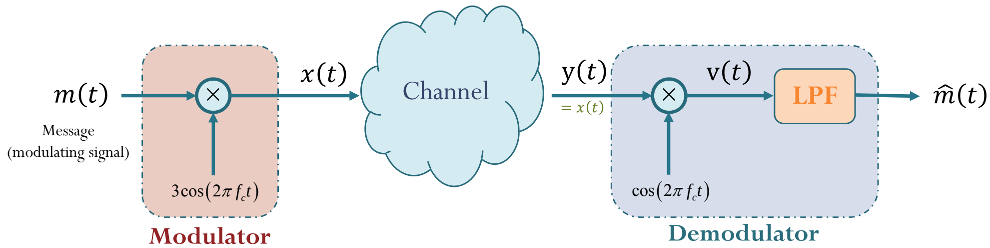

Figure 4.1: DSB-SC modem with no channel impairment

Problem 4. This question starts with a ***square-modulator*** for DSB-SC . Then, the use of the square-operation block is further explored on the receiver side of the system. [Doerschuk, 2008, Cornell ECE 320]

(a) Let $x(t) = A_c m(t)$ where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Consider the block diagram shown in Figure 4.2.

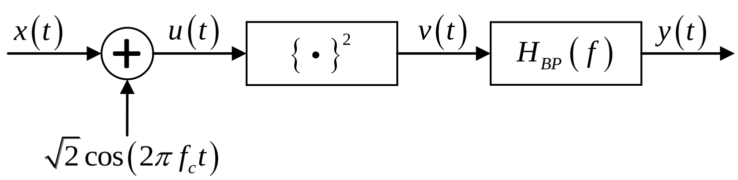

Figure 4.2: Block diagram for Problem 4a

Assume $f_c \gg B$ and

$$H_{BP}(f) = \begin{cases} 1, & |f - f_c| \le B \\ 1, & |f + f_c| \le B \\ 0, & \text{otherwise.} \end{cases}$$

The block labeled "$\{\cdot\}^2$" has output $v(t)$ that is the square of its input $u(t)$:

$$v(t) = u^2(t).$$

Find $y(t)$.

(b) The block diagram in part (a) provides a nice implementation of a modulator because it may be easier to build a squarer than to build a multiplier. Based on the successful use of a squaring operation in the modulator, we decide to use the same squaring operation in the demodulator. Let

$$x(t) = A_c m(t) \sqrt{2} \cos(2\pi f_c t)$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 4.3.

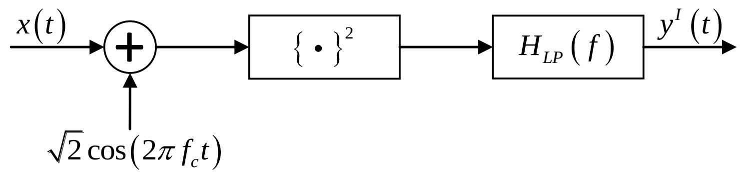

Figure 4.3: Block diagram for Problem 4b

Use

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^I(t)$. Does this block diagram work as a demodulator; that is, is $y^I(t)$ proportional to $m(t)$?

(c) Due to the failure in part (b), we have to think hard and it seems natural to consider also the block diagram with cos replaced by sin. Let

$$x(t) = A_c m(t)\sqrt{2}\cos(2\pi f_c t)$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$ as in part (b). Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 4.4.

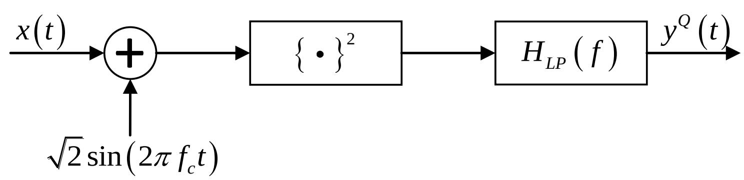

Figure 4.4: Block diagram for Problem 4c

As in part (b), use

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^Q(t)$.

(d) Use the results from parts (b) and (c). Draw a block diagram of a ***successful*** DSB-SC demodulator using squaring operations instead of multipliers.

Problem 5 (Cube modulator)**.** Consider the block diagram shown in Figure 4.5 where "$\{\cdot\}^3$" indicates a device whose output is the cube of its input.

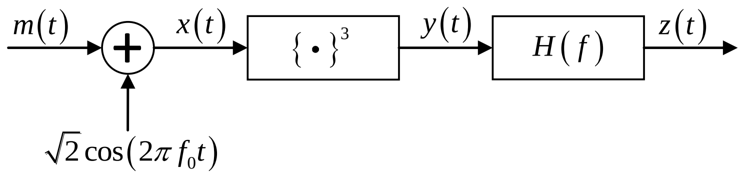

Figure 4.5: Block diagram for Problem 5. Note the use of f_0 instead of f_c.

Let $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ be bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$.

(a) Plot an $H(f)$ that gives $z(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$. What is the gain in $H(f)$? What is the value of f_c? Notice that the frequency of the cosine is f_0 not f_c. You are supposed to determine f_c in terms of f_0.

(b) Let $M(f)$ be

$$M(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

(i) Plot $X(f)$.

(ii) Plot $Y(f)$. Hint:

$$M(f) * M(f) = \begin{cases} 2B - |f|, & |f| \le 2B \\ 0, & \text{otherwise.} \end{cases}$$

Do not attempt to make an accurate plot or calculation for the Fourier transform of $m^3(t)$.

(iii) ~~(c)~~ For your filter of part (a), plot $z(t)$.

[Doerschuk, 2008, Cornell ECE 320]