HW 3 — Due: September 10, 12:59 PM (in class)

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Instructions

- (a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.
- (b) It is important that you try to solve all problems. (5 pt) The extra question at the end is optional.
- (c) Late submission will not be accepted.
- (d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
- (e) For questions that require the use of MATLAB,
 - (i) the codes used should be printed,
 - (ii) if the code is based on some provided code, all modified lines should be highlighted or underlined,
 - (iii) the results displayed in the command window and the figures generated should be captured and printed out as well.

Problem 1 (M2011). The Fourier transform X(f) for a signal x(t) is shown in Figure 3.1.

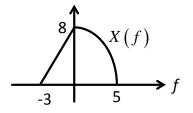


Figure 3.1: Plot of X(f) for Problem 1.

Let y(t) = x(4-2t). Carefully sketch |Y(f)|.

Problem 2. 1

(a) Consider the cosine pulse

$$p(t) = \begin{cases} \cos(10\pi t), & -1 \le t \le 1\\ 0, & \text{otherwise} \end{cases}$$

- (i) Use MATLAB to plot p(t) for $-3 \le t \le 3$.
- (ii) Find P(f) analytically.
- (iii) Use the expression from part (ii) to plot P(f) in MATLAB from -10 Hz to 10 Hz.
- (b) Consider the cosine pulse

$$p(t) = \begin{cases} \cos(10\pi t), & 2 \le t \le 4\\ 0, & \text{otherwise} \end{cases}$$

- (i) Find P(f) analytically.
- (ii) Use MATLAB. Mimic the code in specrect.m to plot the spectrum of p(t). Follow the settings below:
 - Consider the time t from 0 to 10 [s] when you set up the time vector.
 - Use the sampling frequency of 500 samples per sec. So, the sampling interval (the time between adjacent samples) is $T_s = 1/500$.
 - With the above sampling frequency, plotspect will plot the magnitude spectrum from -250 to 250 Hz. Use the function xlim (or the magnifier glass GUI) to limit your frequency view to be only from -10 to +10 Hz.
- (iii) Compare your analytical answer from part (i) with the plot in part (ii). Caution: The built-in sinc function in MATLAB is defined using the normalized version. So, you will need to remove a factor of π from the argument of each sinc function found in part (i) when you type it into MATLAB.

Problem 3. You are given the baseband signals (i) $m(t) = \cos 1000\pi t$; (ii) $m(t) = 2\cos 1000\pi t + \cos 2000\pi t$; (iii) $m(t) = (\cos 1000\pi t) \times (\cos 3000\pi t)$. For each one, do the following.

- (a) Sketch the spectrum of m(t).
- (b) Sketch the spectrum of the DSB-SC signal $m(t) \cos 10,000\pi t$.

[Lathi and Ding, 2009, Q4.2-1]

¹Inspired by [Carlson and Crilly, 2009, Q2.2-1 and Q2.2-2].

Extra Question

Here is an optional question for those who want more practice.

Problem 4 (M2011). Use properties of Fourier transform to evaluate the following integrals. (Do not integrate directly. Recall that $\operatorname{sinc}(x) = \frac{\sin(x)}{x}$.) Clearly state the property or properties that you use.

- (a) $\int_{-\infty}^{\infty} \operatorname{sinc} (\sqrt{5}x) dx$ (b) $\int_{-\infty}^{\infty} e^{-2\pi f \times 2j} 2\operatorname{sinc} (2\pi f) \left(e^{-2\pi f \times 5j} 2\operatorname{sinc} (2\pi f) \right)^* df$ (c) $\int_{-\infty}^{\infty} \operatorname{sinc} (\sqrt{5}x) \operatorname{sinc} (\sqrt{7}x) dx$
- (d) $\int_{-\infty}^{\infty} \operatorname{sinc}\left(\pi\left(x-5\right)\right) \operatorname{sinc}\left(\pi\left(x-\frac{7}{2}\right)\right) dx$