ECS 332: Principles of Communications **2012/1**

HW 3 — Due: Aug 8 and Aug 10

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

(e) [M2011] means the problem is from the 2011 midterm exam which is posted on the course website.

(f) Submit Problems 1-3 by Aug 8 and Problems 4-6 by Aug 10.

Problem 1.

(a) Give a simplified expression for the Fourier transform $P(f)$ of a waveform $p(t)$ when

$$p(t) = \begin{cases} A, & 0 \le t < T \\ 0, & \text{otherwise} \end{cases}$$

(b) A message $m = (m[0], m[1], m[2], m[3]) = (1, -1, 1, 1)$ is sent via

$$x(t) = \sum_{k=0}^{\ell-1} m[k]\, p(t - kT),$$

where ℓ is the length of m.

Find a simplified expression for the Fourier transform $X(f)$ of the waveform $x(t)$.

(c) Assume $T = 2$ [ms] and $A = 1$ [mV]. For $X(f)$ generated by m given below, analytically evaluate $X(0)$.

(i) $m = (1)$;

(ii) $m = (1, 1)$

(iii) $m = (1, 1, 0, 0)$

(iv) $m = (1, 1, -1)$

(v) $m = (1, 1, -1, 1)$

(vi) $m = (1, 1, -1, -1)$

(vii) $m = (1, 1, -1, 1, -1, -1, 1, 1, 1, -1, 1, 1)$

(d) When we know how to find $X(f)$ analytically, we may use its expression to plot $|X(f)|$ in `MATLAB` without the help of `plotspec.m`.

With the help of the provided function `FTofManyShiftedRect.m`, you may run `HW3_Q1.m` to plot $|X(f)|$ from part (b).

Modify the code in `HW3_Q1.m` to plot $|X(f)|$ for the m given in part (c).

(e) What did you learn from the plots in part (d)?

Problem 2 (Fourier Transform of Digital Transmisson)**.** Solve Question 5 in [M2011].

Actually, I meant Question 12 in [M2011]. So, you may choose to do Question 5 or Question 12 for this problem.

Problem 3 (QAM)**.** Let

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t).$$

In class, we have shown that

$$\text{LPF}\left\{x_{\text{QAM}}(t)\sqrt{2}\cos(\omega_c t)\right\} = m_1(t)$$

Give a similar proof to show that

$$\text{LPF}\left\{x_{\text{QAM}}(t)\sqrt{2}\sin(\omega_c t)\right\} = m_2(t)$$

Problem 4. In ***quadrature amplitude modulation*** (***QAM***) or ***quadrature multiplexing***, two baseband signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the following QAM signal:

$$x_{\mathrm{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t).$$

An error in the phase or the frequency of the carrier at the demodulator in QAM will result in loss and interference between the two channels (cochannel interference).

In this problem, show that

$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\cos((\omega_c+\Delta\omega)t+\delta)\right\} = m_1(t)\cos((\Delta\omega)t+\delta) - m_2(t)\sin((\Delta\omega)t+\delta)$$
$$\mathrm{LPF}\left\{x_{\mathrm{QAM}}(t)\sqrt{2}\sin((\omega_c+\Delta\omega)t+\delta)\right\} = m_1(t)\sin((\Delta\omega)t+\delta) + m_2(t)\cos((\Delta\omega)t+\delta).$$

Problem 5. Consider the basic DSB-SC transceiver with time-delay channel presented in class. Recall that the input of the receiver is

$$x(t-\tau) = m(t-\tau)\sqrt{2}\cos(\omega_c(t-\tau))$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

(a) Suppose that, at the receiver, we multiply by $\sqrt{2}\cos((\omega_c t)-\theta)$ instead of $\sqrt{2}\cos(\omega_c t)$ as illustrated in Figure 3.1. Assume

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $\hat{m}(t)$ (the output of the LPF).

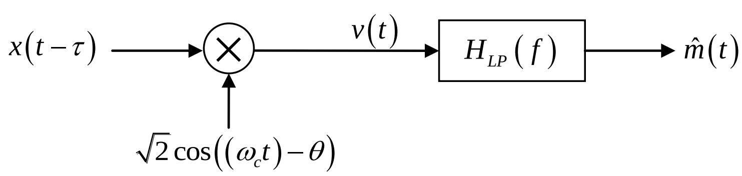

Figure 3.1: Receiver for Problem 5a

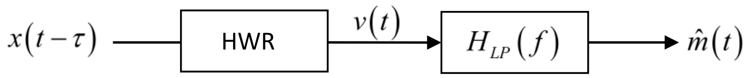

Figure 3.2: Receiver for Problem 5b

(b) Use the same assumptions as part (a). However, at the receiver, instead of multiplying by $\sqrt{2}\cos((\omega_c t) - \theta)$, we pass $x(t-\tau)$ through a half-wave rectifier (HWR) as shown in Figure 3.2.

Make an extra assumption that $m(t) \geq 0$ for all time t and that the half-wave rectifier input-output relation is described by a function $f(\cdot)$:

$$f(x) = \begin{cases} x, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Find $\hat{m}(t)$ (the output of the LPF).

Problem 6. Solve Question 7 in [M2011].