**ECS 332: Principles of Communications** **2012/1**

# HW 6 — Due: Oct 5

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## Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

**Problem 1.** Consider the code $\{0, 01\}$

code book

reversed code

| $x$ | $c(x)$ | $\tilde{c}(x)$ |
|---|---|---|
| ? | 0 | 0 |
| ? | 01 | 10 |

suffix-free code

$\hookrightarrow$ prefix-free

(a) Is it nonsingular? Yes

(b) Is it uniquely decodable? Yes

(c) Is it prefix-free? No

**Problem 2.** Consider the random variable $X$ whose support $S_X$ contains seven values:

$$S_X = \{x_1, x_2, \ldots, x_7\}.$$

Their corresponding probabilities are given by

| $x$ | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ |
|---|---|---|---|---|---|---|---|
| $p_X(x)$ | 0.49 | 0.26 | 0.12 | 0.04 | 0.04 | 0.03 | 0.02 |

(a) Find the entropy $H(X)$. $\sum_x p_X(x) \log_2 \frac{1}{p_X(x)} = -\sum_x p_X(x) \log_2 p_X(x)$

(b) Find a binary Huffman code for $X$.

(c) Find the expected codelength for the encoding in part (b).

**Problem 3.** Find the entropy and the binary Huffman code for the random variable $X$ with pmf

$$p_X(x) = \frac{x}{21} \text{ for } x = 1, 2, \ldots, 6.$$

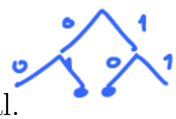
Also calculate $\mathbb{E}[\ell(X)]$ when Huffman code is used.

**Problem 4.** Construct a random variable $X$ (and its pmf) whose Huffman code is $\{0, 10, 11\}$.

**Problem 5.** These codes cannot be Huffman codes. Why?

(a) $\{00, 01, 10, 110\}$

(b) $\{01, 10\}$





Hint: Huffman code is optimal.

**Problem 6.** A memoryless source emits two possible message Y(es) and N(o) with probability 0.9 and 0.1, respectively.

(a) Determine the entropy (per source symbol) of this source.

(b) Find the expected codeword length per symbol of the Huffman binary code for the third-order extensions of this source.

(c) Use `MATLAB` to find the expected codeword length per symbol of the Huffman binary code for the fourth-order extensions of this source.

(d) Use `MATLAB` to plot the expected codeword length per symbol of the Huffman binary code for the $n$th-order extensions of this source for $n = 1, 2, \ldots, 8$.