Q1 Sinc Review

Monday, September 17, 2012
9:09 PM

(a) No, the sinc function is not time-limited.

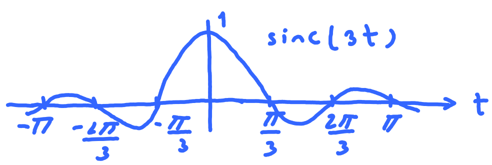
(b) Sinc function in time domain corresponds to rectangular function in frequency domain.

So, yes, it is band-limited.

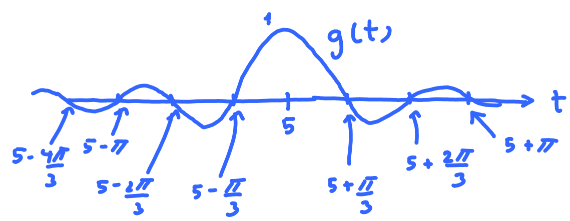
(c) $g(t) = \text{sinc}(3(t-5))$.

First, we plot $\text{sinc}(3t) = \dfrac{\sin(3t)}{3t} \rightarrow = 0$ when $3t = k\pi$

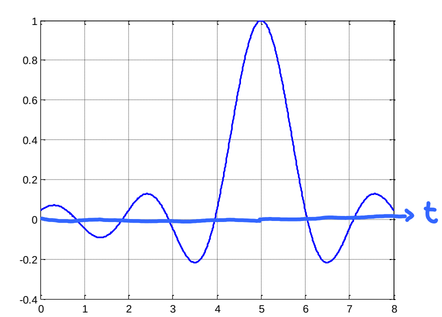
$t = k\frac{\pi}{3}$



Then, for $g(t)$, the graph of $\text{sinc}(3t)$ is shifted to $t = 5$



To check our sketch above, we also provide the plot of $g(t)$ from MATLAB.



(d) $|G(f)|$ is the same as $|X(f)|$ where $x(t) = \text{sinc}(3t)$.

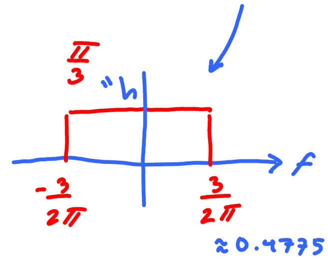
$\text{sinc}(3t) = \dfrac{\sin(3t)}{3t}$ $\quad 3t = 2\pi \times \frac{3}{2\pi} \times t$

Because $x(0) = 1$,

we need

$h \times 2 \times \frac{3}{2\pi} = 1$

$h = \frac{\pi}{3}$



Q2 Reconstruction Formula

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It is easy to search your lecture notes for the reconstruction formula.

The formula is

$$g_r(t) = \sum_{n=-\infty}^{\infty} g[n]\,\mathrm{sinc}(\pi f_s (t - nT_s))$$

sampling rate

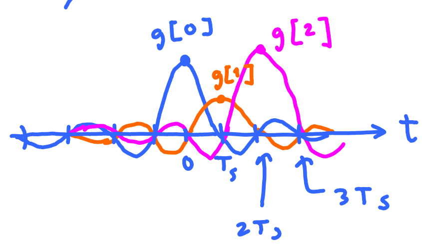
sampling interval.

However, this problem is all about trying to write the formula above by recalling the reconstruction process.

To do this, recall that reconstruction is a process in which we try to reconstruct $g(t)$ from $g[n]$, where

$$g[n] = g(nT).$$

To do this, we simply interpolate the values between the samples by the sinc function:



So, you know that the reconstruction formula must be of the form

$$g_r(t) = \sum_{n=-\infty}^{\infty} g[n]\,\mathrm{sinc}(c(t - nT_s))$$

The constant here is determined by the fact that we want

$$\mathrm{sinc}(ct) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm T_s, \pm 2T_s, \pm 3T_s, \dots \end{cases}$$

So, we need $ckT_s = k\pi$

$$\Downarrow$$

$$c = \frac{\pi}{T_s} = \pi f_s.$$

Q3 Nyquist's Criterion

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(a) In the time domain:

$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm T, \pm 2T, \pm 3T, \dots \end{cases}$$

symbol "duration" → T does not necessarily equal the pulse duration (the pulse may not even be time-limited.)

"interval"

$\frac{1}{T}$ = signaling rate measured in symbols per second or baud

(b) In the freq. domain:

$$\star \quad \sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) \equiv T$$

Reminder: A pulse $p(t)$ is called a Nyquist pulse iff it satisfies $\star$

Q4 Nyquist Pulses

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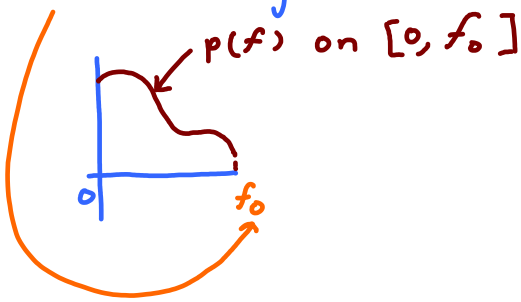
General strategy (recipe)

Let the end of the given interval be $\frac{1}{2T}$.



Then, set the symbol duration to be $T = \frac{1}{2f_0}$.

Our pulse will be band-limited to $\frac{1}{T}$.

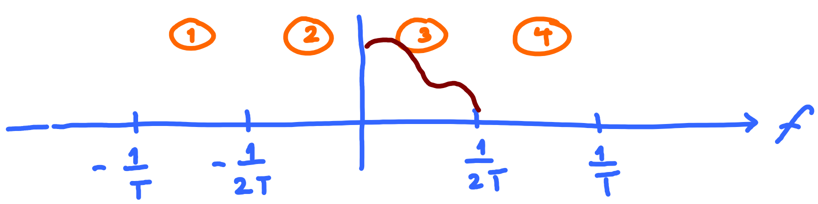
Recall that...

to check whether a pulse is a Nyquist pulse, we check whether

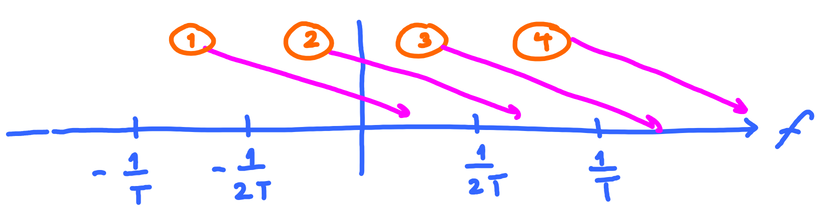
$$\sum_{k=-\infty}^{\infty} P\left(f - \frac{k}{T}\right) \equiv T.$$

P(f) is replicated every $\frac{1}{T}$.

Now consider 4 intervals:



Note that when P(f) is copied to $\frac{1}{T}$, its content in region 1 will show up in region 3



Therefore, when we add P(f) in region 1 and P(f) in region 3, we must have T.

In other words, $P(f)$ in region 1 can be found by $T - P(f)$ in region 3.

The (hinted) suggested symmetry in $P(f)$ allow us to find

$P(f)$ in region 2 by flipping $P(f)$ in region 3 horizontally

and

$P(f)$ in region 4 by flipping $P(f)$ in region 1 horizontally.

For this question, we have $f_0 = 1$. Therefore, we will choose

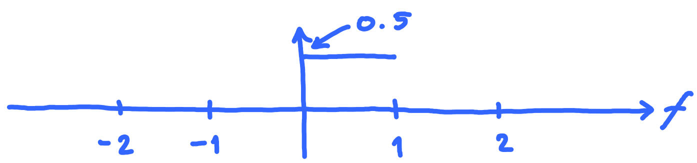
$T = \frac{1}{2f_0} = 0.5. \Rightarrow \frac{1}{T} = 2$ and $\frac{1}{2T} = 1.$

The four regions under consideration are

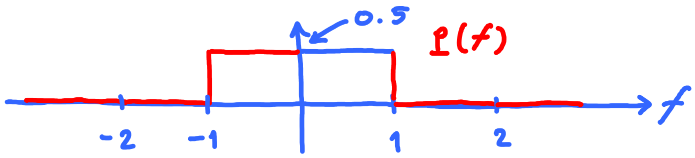
1 $f \in [-2, -1)$
2 $f \in [-1, 0)$
3 $f \in [0, 1)$
4 $f \in [1, 2]$

The question specifies $P(f)$ in region 3 as discussed above.
We will use the strategy described above to find $P(f)$ which is band-limited to $\frac{1}{T}$.

(a)



Note that in region 3, $P(f) \equiv T$ already. Therefore, we need nothing from region 1; in other words, $P(f) \equiv 0$ in region 1



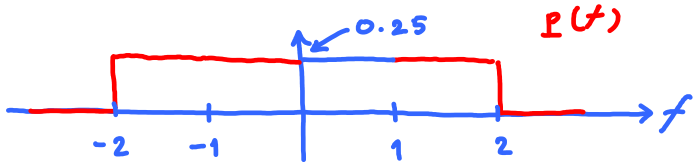
(There is no need to check whether $P(f)$ satisfies the Nyquist criterion again because it is constructed by our recipe above.)

(b) In this part, the value of $P(f)$ in region 3 is 0.25;

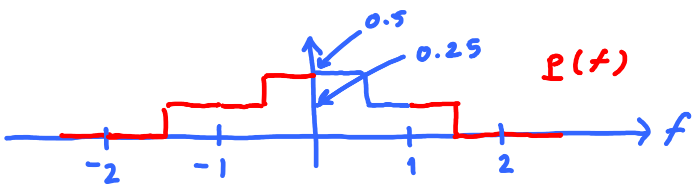
So, we will set the value in region 1 to be

$$T - 0.25 = 0.5 - 0.25 = 0.25.$$

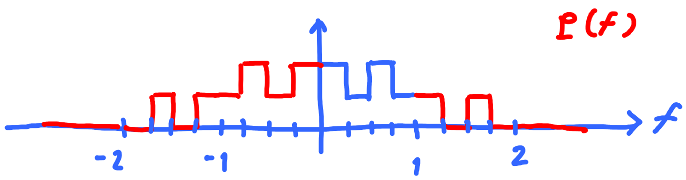
The values in region 2 and 4 are from the symmetry in $\rho(f)$.



(c)



(d)



Q5 Raised Cosine Pulse

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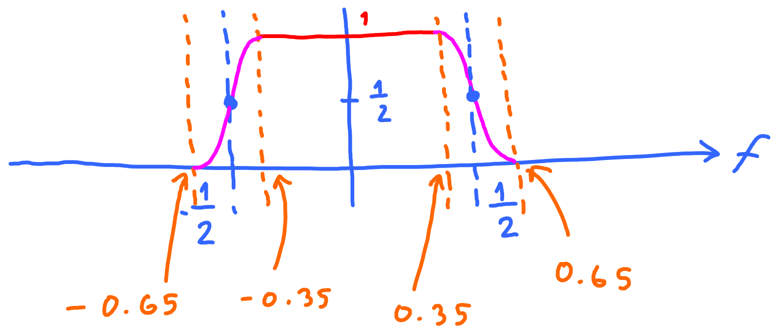
$\alpha = 0.3, \quad T = 1$

(a)

$\frac{1}{2T} = \frac{1}{2}$

$\frac{\alpha}{T} = \frac{0.3}{1} = 0.3$

$\frac{\alpha}{2T} = \frac{0.3}{2} = 0.15$



(b) The raised cosine pulse is a Nyquist pulse.
Therefore,

$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm T, \pm 2T, \pm 3T, \dots \end{cases}$$

Here, $T = 1$.

Hence,
$$p(t) = \begin{cases} 1, & t = 0 \\ 0, & t = \pm 1, \pm 2, \pm 3 \dots \end{cases}$$

In particular, $p(2) = 0$

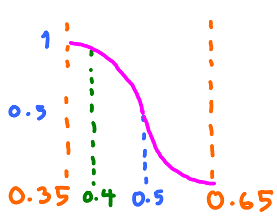
(c) Note that $P\left(\frac{1}{2T}\right) = \frac{T}{2}$.

Here, $T = 1$. Hence, $P\left(\frac{1}{2}\right) = \frac{1}{2}$.

(d) From part (a), we've seen that $P(f) = 1$ for $f \in [-0.35, 0.35]$.

Therefore, $P(0.3) = 1$.

(e) We magnify the plot in part (a) for clarity



f: 0 π (half a cycle)

$\frac{\alpha}{T} = 0.3$

This is the "raised" part of RC.

$$\Rightarrow P(0.4) = \frac{1}{2}\left(1 + \cos\left(\pi \times \frac{0.05}{0.3}\right)\right)$$

$$= \frac{1}{2}\left(1 + \cos\left(\pi/6\right)\right)$$

$$= \frac{2+\sqrt{3}}{4} \approx 0.933$$

Q6 Raised Cosine Pulse

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We have seen in class that the formula for the raised cosine pulse in the time domain is

$$p_{RC}(t) = \frac{\cos\left(\alpha\pi \frac{t}{T}\right)}{1-\left(2\alpha\frac{t}{T}\right)^2} \frac{\sin\left(\pi\frac{t}{T}\right)}{\pi\frac{t}{T}}.$$

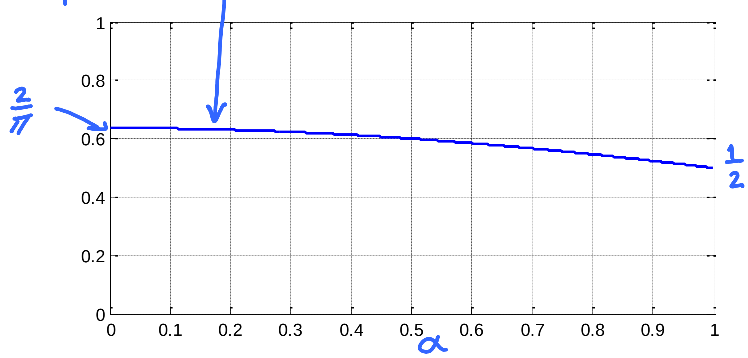
(I still don't have an easy way to "remember" this formula except the fact that it will need to have a factor of $\mathrm{sinc}\left(\pi\frac{t}{T}\right)$.)

At $t = \frac{T}{2}$, we have $\frac{t}{T} = \frac{1}{2}$

$$p_{RC}\left(\frac{T}{2}\right) = \frac{\cos\left(\alpha\frac{\pi}{2}\right)}{1-\alpha^2} \frac{\sin\frac{\pi}{2}}{\frac{\pi}{2}} \rightarrow 1$$

$$= \frac{2}{\pi} \times \frac{\cos\left(\alpha\frac{\pi}{2}\right)}{1-\alpha^2}$$

MATLAB plot:



Note that when $\alpha = 0$, $p_{RC}\left(\frac{T}{2}\right) = \frac{2}{\pi} \approx 0.6366$

As $\alpha \rightarrow 1$, $p_{RC}\left(\frac{T}{2}\right) \rightarrow \cancel{\frac{2}{\pi}} \times \frac{\cancel{\frac{\pi}{2}}\left(-\sin\left(\alpha\frac{\pi}{2}\right)\right)}{-2\alpha}\Bigg|_{\alpha=1} = \frac{1}{2}.$