HW 4 — Due: Sep 7

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Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. The spectrum of a periodic square wave can be found using:

```
% specsquare.m plot the spectrum of a square wave
close all
f0=10;                  % "(fundamental) frequency" of the square wave
EndTime=2;              % Will consider from time = 0 to EndTime
Ts=1/1000;             % sampling interval (time interval between samples)
t=0:Ts:(EndTime-Ts);   % create a time vector
x=sign(cos(2*pi*f0*t)); % square wave = sign of cos wave
plotspec(x,Ts)          % call plotspec to draw spectrum
```

The output of `specsquare.m` is shown in Figure 4.1. The top plot shows the first 2 seconds of a square wave with fundamental frequency $f_0 = 10$ cycles per second. The bottom plot shows a series of spikes that define the frequency content. In this case, the largest spike occurs at ± 10 Hz, followed by smaller spikes at all the odd-integer multiples (i.e., at $\pm 30, \pm 50, \pm 70$, etc.).

Modify `specsquare.m` to investigate the relationship between the time behavior of the square wave and its spectrum. Try square waves with different (fundamental) frequencies: $f_0 = 20, 40, 100, 300$ Hz. (Keep $T_s = 1/1000$.) Describe the aliasing effect on each of cases. How do the time plots change? How do the spectra change?
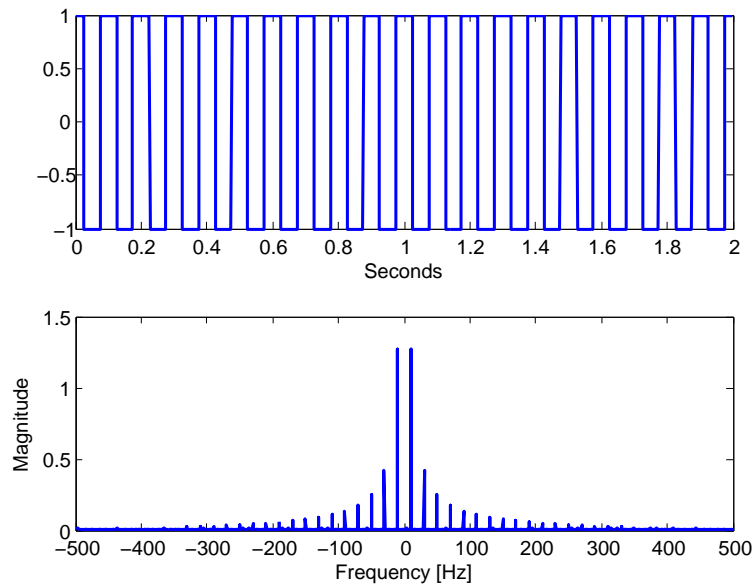
Figure 4.1: Plots from `specsquare.m`

Problem 2. Determine the Nyquist sampling rate and the Nyquist sampling interval for the signals:

(a) $\operatorname{sinc}(100\pi t)$

(b) $\operatorname{sinc}^2(100\pi t)$

(c) $\operatorname{sinc}(100\pi t) + \operatorname{sinc}(50\pi t)$

(d) $\operatorname{sinc}(100\pi t) + 3\operatorname{sinc}^2(60\pi t)$

(e) $\operatorname{sinc}(50\pi t)\operatorname{sinc}(100\pi t)$

Remark: Recall that in our class, $\operatorname{sinc}(x) = \frac{\sin(x)}{x}$.

$\delta t = k\chi$

Problem 3. Consider a signal $g(t) = \operatorname{sinc}(\pi t)$. $= \dfrac{\sin(\pi t)}{\pi t}$

(a) Sketch the Fourier transform $G(f)$ of $g(t)$.

(b) Find the Nyquist sampling rate. $2 \times \frac{1}{2} = 1$ sample/sec.

(c) Recall that the instantaneous sampled signal $g_\delta(t)$ is defined by

$$g_\delta(t) = g(t) \times \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$$

where T_s is the sampling interval.

(i) Let $T_s = 0.5.$ Sketch the Fourier transform $G_\delta(f)$ of $g_\delta(t)$.
(ii) Let $T_s = 4/3$. Sketch the Fourier transform $G_\delta(f)$ of $g_\delta(t)$.

(d) The sequence of sampled values $g[n]$ is constructed from $g(t)$ by

$$g[n] = g(t)|_{t=nT_s}.$$

Recall the reconstruction equation:

$$g_r(t) = \sum_{n=-\infty}^{\infty} g[n] \operatorname{sinc}(\pi f_s(t - nT_s)).$$

Note that we write $g_r(t)$ instead of $g(t)$ to accommodate the case that the sampling rate is too low; in which case, the reconstructed signal is not the same as $g(t)$.

(i) With $T_s = 1$,
   i. Find $g[n]$ for $n = \ldots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \ldots$
   ii. Use the reconstruction equation to find $g_r(t)$. $= \operatorname{sinc}(\pi t)$

(ii) Let's test the reconstruction equation by using MATLAB to plot $g_r(t)$. Note that the sum in the reconstruction equation extends from $-\infty$ to $+\infty$. In MATLAB, we can not add that many terms. So, we need to stop at some n. In this part, use $T_s = 0.5$.

   i. Use MATLAB to plot $g_r(t)$ when only $n = 0$ term is included.
   ii. Use MATLAB to plot $g_r(t)$ and all of its sinc components. Include only $n = -1, 0, 1$.
   iii. Use MATLAB to plot $g_r(t)$ and all of its sinc components. Include only $n = -5, -4, \ldots, -1, 0, 1, \ldots, 4, 5$.
   iv. Use MATLAB to plot $g_r(t)$ and all of its sinc components. Include only $n = -10, -9, \ldots, -1, 0, 1, \ldots, 9, 10$.

   In all these plots, consider t from -4 to 4. Also include the plot of $\operatorname{sinc}(\pi t)$ for comparison.