ECS 332: Principles of Communications **2012/1**

HW 2 — Due: July 27

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems (5 pt). However, Question 3.c, Question 8 and Question 9 are optional.

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. [1] Using `MATLAB` to find the spectrum of a signal:

A signal $g(t)$ can often be expressed in analytical form as a function of time t, and the Fourier transform is defined as the integral of $g(t)\exp(-j2\pi ft)$. Often however, there is no analytical expression for a signal, that is, there is no (known) equation that represents the value of the signal over time. Instead, the signal is defined by measurements of some physical process. For instance, the signal might be the waveform at the input to the receiver, the output of a linear filter, or a sound waveform encoded as an mp3 file.

In all these cases, it is not possible to find the spectrum by analytically performing a Fourier transform. Rather, the discrete Fourier transform (or DFT, and its cousin, the more rapidly computable fast Fourier transform, or FFT) can be used to find the spectrum or frequency content of a measured signal. The `MATLAB` function `plotspec.m`, which plots the spectrum of a signal can be downloaded from our course website. Its help file[2] notes

```
% plotspec(x,Ts) plots the spectrum of the signal x
% Ts = time (in seconds) between adjacent samples in x
```

(a) The function `plotspec.m` is easy to use. For instance, the spectrum of a rectangular pulse $g(t) = 1[0 \le t \le 2]$ can be found using:

[1]Based on [Johnson, Sethares, and Klein, 2011, Sec 3.1 and Q3.3].

[2]You can view the help file for the `MATLAB` function `xxx` by typing `help xxx` at the `MATLAB` prompt. If you get an error such as `xxx` not found, then this means either that the function does not exist, or that it needs to be moved into the same directory as the `MATLAB` application.

```
% specrect.m plot the spectrum of a square wave
close all
time=20;                        % length of time
Ts=1/100;                       % time interval between samples
t=0:Ts:(time-Ts);               % create a time vector
x=[t ≤ 2];                     % rectangular pulse 1[0 ≤ t ≤ 2]
plotspec(x,Ts)                  % call plotspec to draw spectrum
xlim([-5,5])
```

The output of `specrect.m` is shown in Figure 2.1. The top plot shows the first 20 seconds of $g(t)$. The bottom plot shows $|G(f)|$.

Use what we learn in class about the Fourier transform of a rectangular pulse to find a simplified expression for $|G(f)|$. Does your expression agree with the bottom plot in Figure 2.1.

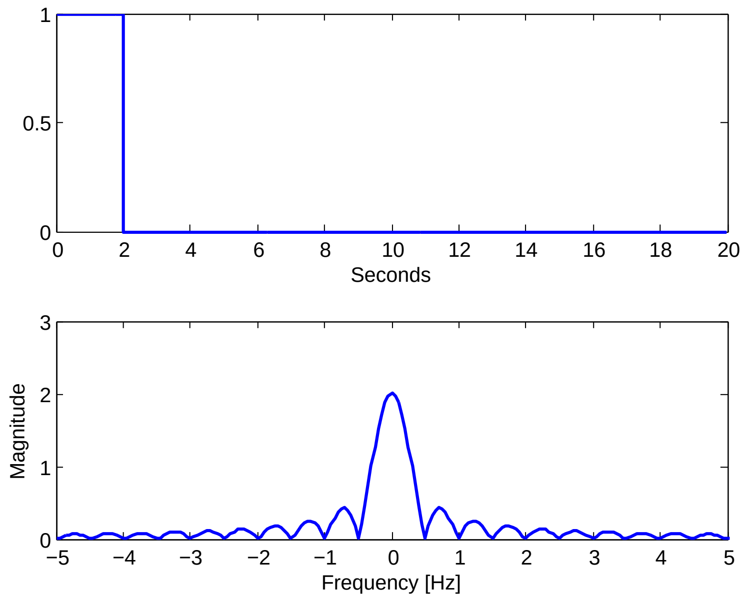

Figure 2.1: Plots from `specrect.m`

(b) Mimic the code in `specsquare.m` to find the spectrum of an exponential pulse

$$s(t) = e^{-t}u(t).$$

Note that you may want to change the parameter `time` to capture most of the content of $s(t)$ in the time domain. You may also use the command `xlim` to "zoom in" the spectrum plot.

(c) Continue from part (b), find $S(f)$ analytically. Compare your analytical answer with the plot in part (b).

(d) `MATLAB` can also perform symbolic manipulation when symbolic toolbox is installed. Run the file `SymbFourier.m`. Check whether you have the same result as part (c).

Problem 2. [3]

(a) Consider the cosine pulse

$$p(t) = \begin{cases} \cos(10\pi t), & -1 \le t \le 1 \\ 0, & \text{otherwise} \end{cases}$$

  (i) Use `MATLAB` to plot $p(t)$ for $-3 \le t \le 3$.

  (ii) Find $P(f)$.

  (iii) Use the expression from part (ii) to plot $P(f)$ in `MATLAB`.

(b) Consider the cosine pulse

$$p(t) = \begin{cases} \cos(10\pi t), & 2 \le t \le 4 \\ 0, & \text{otherwise} \end{cases}$$

  (i) Find $P(f)$ analytically.

  (ii) Use `MATLAB`. Mimic the code in `specsquare.m` to plot the spectrum of $p(t)$.

  (iii) Compare your analytical answer from part (i) with the plot in part (ii).

[3] Inspired by [Carlson and Crilly, 2009, Q2.2-1 and Q2.2-2].

Problem 3. Consider a signal $g(t)$. Recall that $|G(f)|^2$ is called the energy spectral density of $g(t)$. Integrating the energy spectral density over all frequency gives the signal's total energy. The energy contained in the frequency band B can be found from the integral $\int_B |G(f)|^2 df$. In particular, if the band is simply an interval of frequency from f_1 to f_2, then the energy contained in this band is given by

$$\int_{f_1}^{f_2} |G(f)|^2 df. \tag{2.1}$$

In this problem, assume

$$g(t) = 1[-1 \le t \le 1].$$

(a) Find the (total) energy of $g(t)$.

(b) It is mentioned in class that the main lope of the sinc waveform contains about 90% of the total energy. Check this fact by first computing the energy contained in the frequency band occupied by the main lope and then compare with your answer from part (a).

Hint: Find the zeros of the main lope. This give f_1 and f_2. Now, we can apply (2.1). `MATLAB` or similar tools can then be used to numerically evaluate the integral.

(c) Suppose we want to include more energy by considering wider frequency band. Let this band be the interval $B = [-f_0, f_0]$. Find the minimum value of f_0 that allows the band to capture at least 99% of the total energy in $g(t)$.

Problem 4. Given a system with input-output relationship of

$$y = f(x) = 2x + 10,$$

is this system linear? [Carlson and Crilly, 2009, Q2.3-10]

Problem 5. Signal $x(t) = 10\cos(2\pi \times 7 \times 10^6 \times t)$ is transmitted to some destination. The received signal is $y(t) = 10\cos(2\pi \times 7 \times 10^6 \times t - \pi/6)$.

(a) What is the minimum distance between the source and destination?

(b) What are the other possible distances?

[Carlson and Crilly, 2009, Q2.3-14]

Problem 6. You are given the baseband signals (i) $m(t) = \cos 1000\pi t$; (ii) $m(t) = 2\cos 1000\pi t + \cos 2000\pi t$; (iii) $m(t) = (\cos 1000\pi t) \times (\cos 3000\pi t)$. For each one, do the following.

(a) Sketch the spectrum of $m(t)$.

(b) Sketch the spectrum of the DSB-SC signal $m(t)\cos 10{,}000\pi t$.

[Lathi and Ding, 2009, Q4.2-1]

Problem 7. This question starts with a ***square-modulator*** for DSB-SC similar to the one we discussed in class. Then, the use of the square-operation block is further explored on the receiver side of the system. [Doerschuk, 2008, Cornell ECE 320]

(a) Let $x(t) = A_c m(t)$ where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Consider the block diagram shown in Figure 2.2.

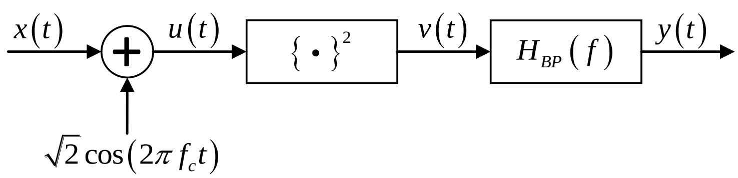

Figure 2.2: Block diagram for Problem 7a

Assume $f_c \gg B$ and

$$H_{BP}(f) = \begin{cases} 1, & |f - f_c| \le B \\ 1, & |f + f_c| \le B \\ 0, & \text{otherwise.} \end{cases}$$

The block labeled "$\{\cdot\}^2$" has output $v(t)$ that is the square of its input $u(t)$:

$$v(t) = u^2(t).$$

Find $y(t)$.

(b) The block diagram in part (a) provides a nice implementation of a modulator because it may be easier to build a squarer than to build a multiplier. Based on the successful use of a squaring operation in the modulator, we decide to use the same squaring operation in the demodulator. Let

$$x(t) = A_c m(t)\sqrt{2}\cos(2\pi f_c t)$$

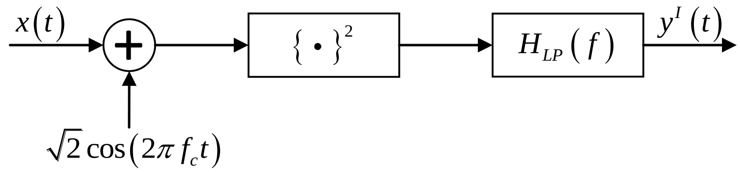

Figure 2.3: Block diagram for Problem 7b

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$. Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 2.3.

Use

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^I(t)$. Does this block diagram work as a demodulator; that is, is $y^I(t)$ proportional to $m(t)$?

(c) Due to the failure in part (b), we have to think hard and it seems natural to consider also the block diagram with cos replaced by sin. Let

$$x(t) = A_c m(t) \sqrt{2} \cos(2\pi f_c t)$$

where $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ is bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$ as in part (b). Again, assume $f_c \gg B$ Consider the block diagram shown in Figure 2.4.

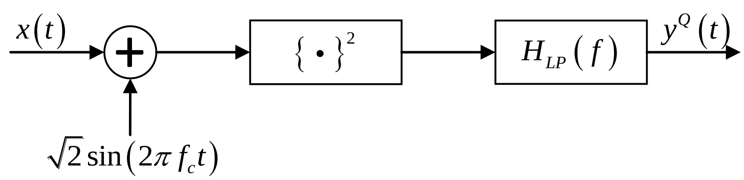

Figure 2.4: Block diagram for Problem 7c

As in part (b), use

$$H_{LP}(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

Find $y^Q(t)$.

(d) Use the results from parts (b) and (c). Draw a block diagram of a ***successful*** DSB-SC demodulator using squaring operations instead of multipliers.

Problem 8 (Cube modulator)**.** Consider the block diagram shown in Figure 2.5 where "$\{\cdot\}^3$" indicates a device whose output is the cube of its input.

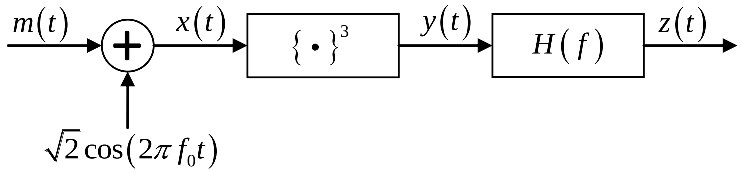

Figure 2.5: Block diagram for Problem 8. Note the use of f_0 instead of f_c.

Let $m(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} M(f)$ be bandlimited to B, i.e., $|M(f)| = 0$ for $|f| > B$.

(a) Plot an $H(f)$ that gives $z(t) = m(t)\sqrt{2}\cos(2\pi f_c t)$. What is the gain in $H(f)$? What is the value of f_c? Notice that the frequency of the cosine is f_0 not f_c. You are supposed to determine f_c in terms of f_0.

(b) Let $M(f)$ be

$$M(f) = \begin{cases} 1, & |f| \le B \\ 0, & \text{otherwise.} \end{cases}$$

(i) Plot $X(f)$.

(ii) Plot $Y(f)$. Hint:

$$M(f) * M(f) = \begin{cases} 2B - |f|, & |f| \le 2B \\ 0, & \text{otherwise.} \end{cases}$$

Do not attempt to make an accurate plot or calculation for the Fourier transform of $m^3(t)$.

(c) For your filter of part (a), plot $z(t)$.

[Doerschuk, 2008, Cornell ECE 320]

Problem 9. You are asked to design a DSB-SC modulator to generate a modulated signal $km(t)\cos(w_c + \theta)$, where $m(t)$ is a signal band-limited to B Hz. Figure 2.6 shows a DSB-SC modulator available in the stockroom. The carrier generator available generates not $\cos \omega_c t$, but $\cos^3 \omega_c t$. Explain whether you would be able to generate the desired signal using only this equipment. You may use any kind of filter you like.

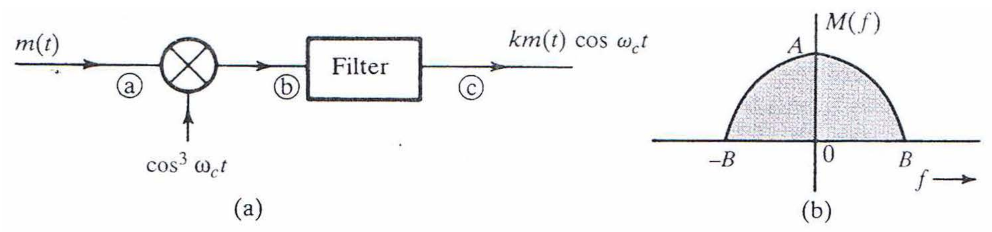

Figure 2.6: Problem 9

(a) What kind of filter is required in Figure 2.6?

(b) Determine the signal spectra at points (b) and (c), and indicate the frequency bands occupied by these spectra.

(c) What is the minimum usable value of f_c?

(d) Would this scheme work if the carrier generator output were $\cos^2 \omega_c t$? Explain.

(e) Would this scheme work if the carrier generator output were $\cos^n \omega_c t$ for any integer $n \geq 2$?

[Lathi and Ding, 2009, Q4.2-3]