ECS 332: Principles of Communications **2012/1**

HW 1 — Due: July 13

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) ONE part of a question will be graded (5 pt). Of course, you do not know which part will be selected; so you should work on all of them.

(b) It is important that you try to solve all problems. (5 pt)

(c) Late submission will be heavily penalized.

(d) Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1. In class, we have seen how to use the Euler's formula to show that

$$\cos^2 x = \frac{1}{2}\left(\cos\left(2x\right) + 1\right).$$

For this question, use similar technique to show that

$$\cos A \cos B = \frac{1}{2}\left(\cos\left(A+B\right) + \cos\left(A-B\right)\right).$$

Problem 2. Listen to the Fourier's Song (Fouriers_Song.mp3) which can be downloaded from

http://sethares.engr.wisc.edu/mp3s/Fouriers_Song.mp3

Which properties of the Fourier Transform can you recognize from the song? List them here.

Problem 3. Derive and plot the signal $x(t)$ whose Fourier transform is given by

$$X\left(f\right) = \operatorname{sinc}^2\left(5\pi f\right) = \left(\frac{\sin\left(5\pi f\right)}{\left(5\pi f\right)}\right)^2.$$

Problem 4. For the signal $g(t)$ shown in Figure 1.1, sketch the signals:

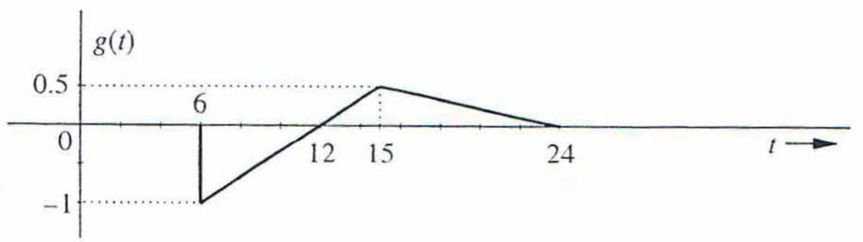

Figure 1.1: Problem 4

(a) $g(-t)$

(b) $g(t+6)$

(c) $g(3t)$

(d) $g(6-t)$.

Problem 5. Evaluate the following integrals:

(a) $\int\limits_{-\infty}^{\infty} g(\tau)\,\delta(t-\tau)d\tau$

(b) $\int\limits_{-\infty}^{\infty} \delta(\tau)\,g(t-\tau)d\tau$

(c) $\int\limits_{-\infty}^{\infty} \delta(t)\,e^{-j2\pi ft}dt$

(d) $\int\limits_{-\infty}^{\infty} \delta(t-2)\sin(\pi t)dt$

(e) $\int\limits_{-\infty}^{\infty} \delta(t+3)\,e^{-t}dt$

(f) $\int\limits_{-\infty}^{\infty} (t^3+4)\,\delta(1-t)dt$

(g) $\int\limits_{-\infty}^{\infty} g(2-t)\,\delta(3-t)dt$

(h) $\int_{-\infty}^{\infty} e^{(x-1)} \cos\left(\frac{\pi}{2}(x-5)\right) \delta(x-3)dx$

Problem 6. The Fourier transform of the triangular pulse $g(t)$ in Figure 1.2a is given as

$$G(f) = \frac{1}{(2\pi f)^2}\left(e^{j2\pi f} - j2\pi f e^{j2\pi f} - 1\right)$$

Using this information, and the time-shifting and time scaling properties, find the Fourier transforms of the signals shown in Figure 1.2b, c, d, e, and f.

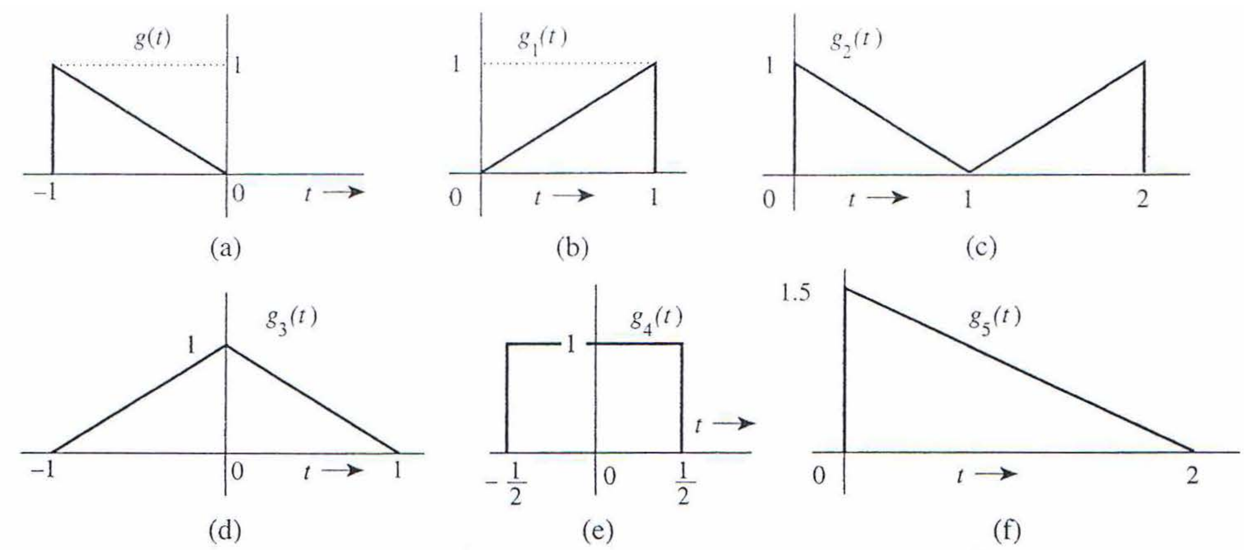

Figure 1.2: Problem 6

Problem 7. Use properties of Fourier transform to evaluate the following integrals. (Do not integrate directly. Recall that $\operatorname{sinc}(x) = \frac{\sin(x)}{x}$.) Clearly state the property or properties that you use.

(a) $\int\limits_{-\infty}^{\infty} \operatorname{sinc}\left(\sqrt{5}x\right) dx$

(b) $\int\limits_{-\infty}^{\infty} e^{-2\pi f \times 2j} 2\operatorname{sinc}\left(2\pi f\right) \left(e^{-2\pi f \times 5j} 2\operatorname{sinc}\left(2\pi f\right)\right)^{*} df$

(c) $\int\limits_{-\infty}^{\infty} \operatorname{sinc}\left(\sqrt{5}x\right) \operatorname{sinc}\left(\sqrt{7}x\right) dx$

(d) $\int\limits_{-\infty}^{\infty} \operatorname{sinc}\left(\pi\left(x-5\right)\right) \operatorname{sinc}\left(\pi\left(x-\frac{7}{2}\right)\right) dx$