

ECS 332: In-Class Exercise # 8 - Sol

Instructions

1. Separate into groups of no more than three students each.
The group cannot be the same as any of your former groups.
2. Explanation is not required for this exercise [ENRE]
3. **Do not panic.**

Date: 18 / 09 / 2019			
Name			ID <small>(last 3 digits)</small>
Prapun			5 5 5

1. The impulse response of a multipath channel is of the form

$$h(t) = \sum_{k=1}^{\nu} \beta_k \delta(t - \tau_k).$$

a. Suppose $\nu = 2$, $\beta_1 = \beta_2 = 0.5$, $\tau_1 = 1$, $\tau_2 = 3$.

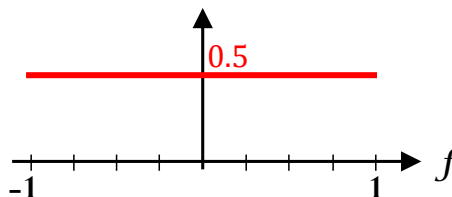
For each of the following channel input $x(t)$, find the corresponding channel output $y(t)$.

Note that the output should be of the form $y(t) = A \cos(2\pi f_0 t + \theta_0)$ for some constants A , f_0 , and θ_0 .

Channel input	Channel output
$x(t) = \cos(\pi t)$	$ \begin{aligned} y(t) &= 0.5x(t-1) + 0.5x(t-3) \\ &= 0.5 \cos(\pi(t-1)) + 0.5 \cos(\pi(t-3)) \\ &= 0.5 \cos(\pi t - \pi) + 0.5 \cos(\pi t - 3\pi) \\ &= -0.5 \cos(\pi t) - 0.5 \cos(\pi t) \\ &= \mathbf{-\cos(\pi t)} \end{aligned} $
$x(t) = \cos\left(\frac{\pi}{2}t\right)$	$ \begin{aligned} y(t) &= 0.5x(t-1) + 0.5x(t-3) \\ &= 0.5 \cos\left(\frac{\pi}{2}(t-1)\right) + 0.5 \cos\left(\frac{\pi}{2}(t-3)\right) \\ &= 0.5 \cos\left(\frac{\pi}{2}t - \frac{\pi}{2}\right) + 0.5 \cos\left(\frac{\pi}{2}t - \frac{3\pi}{2}\right) \\ &\text{Conversion to phasor form } \Leftrightarrow 0.5 \angle -90^\circ + 0.5 \angle -270^\circ = 0 \\ &\text{Conversion back to time domain } \Leftrightarrow 0 \cos\left(\frac{\pi}{2}t + 0\right) \equiv \mathbf{0} \end{aligned} $

b. Suppose $\nu = 1$, $\beta_1 = 0.5$, $\tau_1 = 3$.

Plot $|H(f)|$ from $f = -1$ to $f = 1$ Hz.



When $\nu = 1$, we have $h(t) = \beta_1 \delta(t - \tau_1)$. With the provided values, we have

$$h(t) = 0.5 \delta(t - 3).$$

Therefore, $H(f) = 0.5 e^{-j2\pi 3f}$ and $|H(f)| = 0.5 |e^{-j6\pi f}| \equiv 0.5 \times 1 = \mathbf{0.5}$.

Note that this is a distortionless channel. So, the magnitude spectrum should be flat.