Problem 5. In ***quadrature amplitude modulation*** (***QAM***) or ***quadrature multiplexing***, two baseband signals $m_1(t)$ and $m_2(t)$ are transmitted simultaneously via the following QAM signal:

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(\omega_c t) + m_2(t)\sqrt{2}\sin(\omega_c t).$$

An error in the phase or the frequency of the carrier at the demodulator in QAM will result in loss and interference between the two channels (cochannel interference).

In this problem, show that

$$\text{LPF}\left\{x_{\text{QAM}}(t)\sqrt{2}\cos((\omega_c+\Delta\omega)t+\delta)\right\} = m_1(t)\cos((\Delta\omega)t+\delta) - m_2(t)\sin((\Delta\omega)t+\delta)$$
$$\text{LPF}\left\{x_{\text{QAM}}(t)\sqrt{2}\sin((\omega_c+\Delta\omega)t+\delta)\right\} = m_1(t)\sin((\Delta\omega)t+\delta) + m_2(t)\cos((\Delta\omega)t+\delta).$$

Problem 6. In QAM system, the transmitted signal is of the form

$$x_{\text{QAM}}(t) = m_1(t)\sqrt{2}\cos(2\pi f_c t) + m_2(t)\sqrt{2}\sin(2\pi f_c t).$$

We want to express x_{QAM} in the form

$$x_{\text{QAM}}(t) = \sqrt{2}E(t)\cos(2\pi f_c t + \phi(t)),$$

where $E(t) \geq 0$ and $\phi(t) \in (-180°, 180°]$.

In each part below, we consider different examples of the messages $m_1(t)$ and $m_2(t)$.

(a) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = \sin(2\pi Bt)$ where $0 < B \ll f_c$. Find $E(t)$ and $\phi(t)$.
Hint: $\cos\alpha\cos\beta + \sin\alpha\sin\beta = \cos(\alpha - \beta)$

(b) Suppose $m_1(t) = \cos(2\pi Bt)$ and $m_2(t) = 2\sin(2\pi Bt)$. Let $f_c = 5$ and $B = 2$. Use `MATLAB` to plot the corresponding $E(t)$ and $\phi(t)$ from $t = 0$ to $t = 5$ [sec]. (Hint: the function `angle` or `atan2` will be helpful here.)

Problem 7. Consider a complex-valued signal $x(t)$ whose Fourier transform is $X(f)$.

(a) Find and simplify the Fourier transform of $x^*(t)$.

(b) Find and simplify the Fourier transform of $\mathrm{Re}\{x(t)\}$.

- Hint: $x(t) + x^*(t) = ?$

Problem 8. Consider a (complex-valued) baseband signal $x_b(t) \underset{\mathcal{F}^{-1}}{\overset{\mathcal{F}}{\rightleftharpoons}} X_b(f)$ which is band-limited to B, i.e., $|X_b(f)| = 0$ for $|f| > B$. We also assume that $f_c \gg B$.

(a) The passband signal $x_p(t)$ is given by

$$x_p(t) = \sqrt{2}\,\mathrm{Re}\left\{e^{j2\pi f_c t} x_b(t)\right\}.$$

Find and simplify the Fourier transform of $x_p(t)$.

(b) Find and simplify

$$\mathrm{LPF}\left\{\sqrt{2}\left(\underbrace{\sqrt{2}\,\mathrm{Re}\left\{e^{j2\pi f_c t}x_b\left(t\right)\right\}}_{x_p(t)}\right)e^{-j2\pi f_c t}\right\}.$$

Assume that the frequency response of the LPF is given by

$$H_{LP}\left(f\right)=\begin{cases}1, & |f|\le B\\ 0, & \text{otherwise.}\end{cases}$$