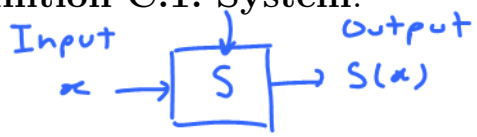
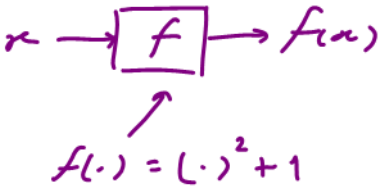
C Linear System

Ex. Function f

Definition C.1. System:





Definition C.2. A **linear system** is a system whose output is linearly related (or directly proportional) to its input[28]. In particular, when we says that the input and output are linearly related, we mean they need to satisfies two properties:

(a) Homogeneous (Scaling): If the input is multiplied by a constant k, then we should observed that the output is also multiplied by k.

$S(kx) = kS(x)$ → $S(c_1x_1 + c_2x_2) = c_1S(x_1) + c_2S(x_2)$

(b) Additive: If the inputs are summed then the output are summed.

$S(x_1 + x_2) = S(x_1) + S(x_2)$

the system whose input-output relationship is governed by

Example C.3. Is the function $f(x) = x^2 + 1$ linear?

check ① $f(kx) \overset{?}{=} kf(x)$ for any k, x

$k^2x^2 + 1 \neq k(x^2+1) = kx^2 + k$ Fail

⇒ not linear!

the system whose input-output relationship is governed by

Example C.4. Is the function $f(x) = 3x + 1$ linear?

① $f(kx) \overset{?}{=} kf(x)$

$3kx + 1 \neq 3kx + k$ Fail

Not linear!

affine

C.5. Any **one-dimensional linear function** can be written in the form

$$y = ax$$

for some constant a.

[28] The input and output are sometimes referred to as cause and effect, respectively.

- For a system, we may call it a **single-input single-output (SISO)** system.
- In radio it is the use of only one antenna both in the transmitter and receiver.

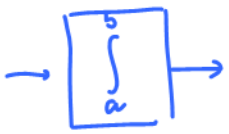
C.6. Any **multi-dimensional linear function** can be written in the form

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{pmatrix} = \mathbf{A} \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{pmatrix}$$

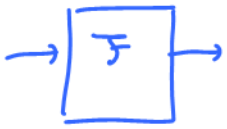
for some matrix $\mathbf{A}$.

- For a system, when both m and n are greater than one, we may call it a **multiple-input multiple-output system (MIMO)** system.

- When $m = n = 1$, we are back to the one-dimensional case in C.5.



$$\int_a^b (c_1 f_1(x) + c_2 f_2(x))\,dx = c_1 \int_a^b f_1(x)\,dx + c_2 \int_a^b f_2(x)\,dx$$



$$\mathcal{F}\{c_1 g_1 + c_2 g_2\} = c_1 \mathcal{F}\{g_1\} + c_2 \mathcal{F}\{g_2\}$$