

ECS 315: Probability and Random Processes**2019/1**

HW 8 — Due: October 31, 4 PM

*Lecturer: Prapun Suksompong, Ph.D.***Instructions**

- (a) This assignment has 4 pages.
- (b) (1 pt) Hard-copies are distributed in class. Original pdf file can be downloaded from the course website. Work and write your answers directly on the provided hardcopy/file (not on other blank sheet(s) of paper).
- (c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page. Furthermore, for online submission, your file name should start with your 10-digit student ID, followed by a space, the course code, a space, and the assignment number: “5565242231 315 HW8.pdf”
- (d) (8 pt) Try to solve all problems.
- (e) Late submission will be heavily penalized.

Problem 1. [F2013/1] For each of the following random variables, find $P[1 < X \leq 2]$.(a) $X \sim \text{Binomial}(3, 1/3)$ (b) $X \sim \text{Poisson}(3)$ **Problem 2.** Arrivals of customers at the local supermarket are modeled by a Poisson process with a rate of $\lambda = 2$ customers per minute. Let M be the number of customers arriving between 9:00 and 9:05. What is the probability that $M < 2$?

Problem 3. [M2011/1] The cdf of a random variable X is plotted in Figure 8.1.

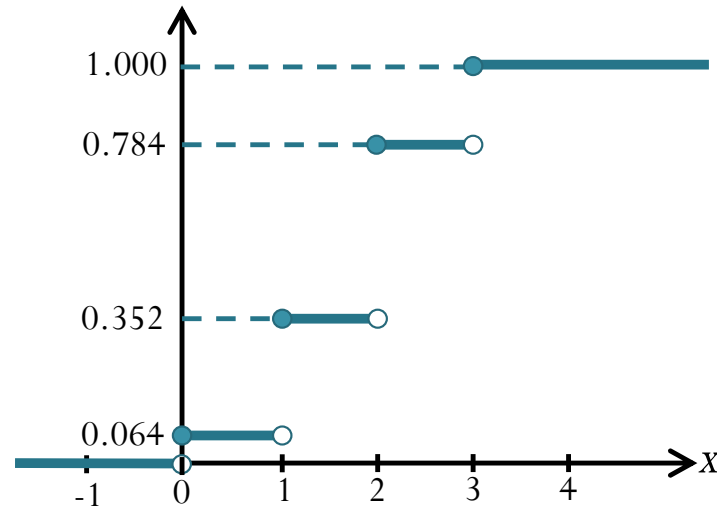


Figure 8.1: CDF of X for Problem 3

(a) Find the pmf $p_X(x)$.

(b) Find the family to which X belongs. (Uniform, Bernoulli, Binomial, Geometric, Poisson, etc.)

Problem 4. When n is large, binomial distribution $\text{Binomial}(n, p)$ becomes difficult to compute directly. In this question, we will consider an approximation when the value of p

is close to 0. In such case, the binomial can be approximated¹ by the Poisson distribution with parameter $\alpha = np$. For this approximation to work, we will see in this exercise that n does not have to be very large and p does not need to be very small.

(a) Let $X \sim \text{Binomial}(12, 1/36)$. (For example, roll two dice 12 times and let X be the number of times a double 6 appears.) Evaluate $p_X(x)$ for $x = 0, 1, 2$.

(b) Compare your answers part (a) with its Poisson approximation.

Problem 5. You go to a party with 500 guests. What is the probability that exactly one other guest has the same birthday as you? Calculate this exactly and also approximately by using the Poisson pmf. (For simplicity, exclude birthdays on February 29.) [Bertsekas and Tsitsiklis, 2008, Q2.2.2]

¹More specifically, suppose X_n has a binomial distribution with parameters n and p_n . If $p_n \rightarrow 0$ and $np_n \rightarrow \alpha$ as $n \rightarrow \infty$, then

$$P[X_n = k] \rightarrow e^{-\alpha} \frac{\alpha^k}{k!}.$$

Extra Questions

Here are some optional questions for those who want more practice.

Problem 6. A sample of a radioactive material emits particles at a rate of 0.7 per second. Assuming that these are emitted in accordance with a Poisson distribution, find the probability that in one second

- (a) exactly one is emitted,
- (b) more than three are emitted,
- (c) between one and four (inclusive) are emitted

[Applebaum, 2008, Q5.27].

Problem 7 (M2011/1). You are given an unfair coin with probability of obtaining a heads equal to $1/3,000,000,000$. You toss this coin $6,000,000,000$ times. Let A be the event that you get “tails for all the tosses”. Let B be the event that you get “heads for all the tosses”.

- (a) Approximate $P(A)$.
- (b) Approximate $P(A \cup B)$.