Name ________________________ ID3 _ _ _

ECS 315: Probability and Random Processes **2019/1**

HW 7 — Due: October 24, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 5 pages.

(b) (1 pt) Hard-copies are distributed in class. Original pdf file can be downloaded from the course website. Work and write your answers **directly on the provided hardcopy/file** (not on other blank sheet(s) of paper).

(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page. Furthermore, for online submission, your file name should start with your 10-digit student ID, followed by a space, the course code, a space, and the assignment number: "5565242231 315 HW4.pdf"

(d) (8 pt) Try to solve all problems.

(e) Late submission will be heavily penalized.

Problem 1. For each description of a random variable X below, indicate whether X is a **discrete** random variable.

(a) X is the number of websites visited by a randomly chosen software engineer in a day.

(b) X is the number of classes a randomly chosen student is taking.

(c) X is the average height of the passengers on a randomly chosen bus.

(d) A game involves a circular spinner with eight sections labeled with numbers. X is the amount of time the spinner spins before coming to a rest.

(e) X is the thickness of the longest book in a randomly chosen library.

(f) X is the number of keys on a randomly chosen keyboard.

(g) X is the length of a randomly chosen person's arm.

Problem 2 (Quiz4, 2014)**.** Consider a random experiment in which you roll a 20-sided fair dice. We define the following random variables from the outcomes of this experiment:

$$X(\omega) = \omega, \quad Y(\omega) = (\omega - 5)^2, \quad Z(\omega) = |\omega - 5| - 3$$

Evaluate the following probabilities:

(a) $P[X = 5]$

(b) $P[Y = 16]$

(c) $P[Y > 10]$

(d) $P[Z > 10]$

(e) $P[5 < Z < 10]$

Problem 3. Consider the sample space $\Omega = \{-2, -1, 0, 1, 2, 3, 4\}$. Suppose that $P(A) = |A|/|\Omega|$ for any event $A \subset \Omega$. Define the random variable $X(\omega) = \omega^2$. Find the probability mass function of X.

Problem 4. Suppose X is a random variable whose pmf at $x = 0, 1, 2, 3, 4$ is given by $p_X(x) = \frac{2x+1}{25}$.

Remark: Note that the statement above does not specify the value of the $p_X(x)$ at the value of x that is not 0,1,2,3, or 4.

(a) What is $p_X(5)$?

(b) Determine the following probabilities:

(i) $P[X = 4]$

(ii) $P[X \le 1]$

(iii) $P[2 \le X < 4]$

(iv) $P[X > -10]$

Problem 5. The random variable V has pmf

$$p_V(v) = \begin{cases} cv^2, & v = 1, 2, 3, 4, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant c.

(b) Find $P[V \in \{u^2 : u = 1, 2, 3, \ldots\}]$.

(c) Find the probability that V is an even number.

(d) Find $P[V > 2]$.

(e) Sketch $p_V(v)$.

(f) Sketch $F_V(v)$. (Note that $F_V(v) = P[V \leq v]$.)

Problem 6. The thickness of the wood paneling (in inches) that a customer orders is a random variable with the following cdf:

$$F_X(x) = \begin{cases} 0, & x < \frac{1}{8}, \\ 0.2, & \frac{1}{8} \le x < \frac{1}{4}, \\ 0.9, & \frac{1}{4} \le x < \frac{3}{8}, \\ 1 & x \ge \frac{3}{8}. \end{cases}$$

Determine the following probabilities:

(a) $P[X \le 1/18]$

(b) $P[X \le 1/4]$

(c) $P[X \le 5/16]$

(d) $P[X > 1/4]$

(e) $P[X \le 1/2]$

[Montgomery and Runger, 2010, Q3-42]

Remark: Try to calculate these values directly from the cdf. (Avoid converting the cdf to pmf first.)