ECS 315: Probability and Random Processes	2019/1

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HW 10 — Due: November 14, 4 PM

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Instructions

(a) This assignment has 4 pages.

(b) (1 pt) Hard-copies are distributed in class. Original pdf file can be downloaded from the course website. Work and write your answers **directly on the provided hardcopy/file** (not on other blank sheet(s) of paper).

(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page. Furthermore, for online submission, your file name should start with your 10-digit student ID, followed by a space, the course code, a space, and the assignment number: "5565242231 315 HW10.pdf"

(d) (8 pt) It is important that you try to solve all problems.

(e) Late submission will be heavily penalized.

Problem 1 (Yates and Goodman, 2005, Q3.2.1)**.** The random variable X has probability density function

$$f_X(x) = \begin{cases} cx & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Use the pdf to find the following quantities.

(a) the constant c

(b) $P[0 \le X \le 1]$

(c) $P\left[-1/2 \leq X \leq 1/2\right]$.

(d) the cdf $F_X(x)$.

Problem 2 (Modified from Yates and Goodman, 2005, Q3.1.3)**.** The CDF of a random variable W is

$$F_W(w) = \begin{cases} 0, & w < -5, \\ (w+5)/8, & -5 \leq w < -3, \\ 1/4, & -3 \leq w < 3, \\ 1/4 + 3\left(w-3\right)/8, & 3 \leq w < 5, \\ 1, & w \geq 5. \end{cases}$$

(a) Is W a continuous random variable?

(b) What is $P\left[W \leq 4\right]$?

(c) What is $P\left[-2 < W \leq 2\right]$?

(d) What is $P\left[W > 0\right]$?

(e) What is the value of a such that $P\left[W \leq a\right] = 1/2$?

Problem 3 (Yates and Goodman, 2005, Q3.2.3). The CDF of random variable W is

$$F_W(w) = \begin{cases} 0, & w < -5, \\ (w+5)/8, & -5 \leq w < -3, \\ 1/4, & -3 \leq w < 3, \\ 1/4 + 3(w-3)/8, & 3 \leq w < 5, \\ 1, & w \geq 5. \end{cases}$$

Find its pdf $f_W(w)$.

Problem 4 (Yates and Goodman, 2005, Q3.3.4). The pdf of random variable Y is

$$f_Y(y) = \begin{cases} y/2 & 0 \leq y < 2, \\ 0, & \text{otherwise.} \end{cases}$$

(a) Find $\mathbb{E}\left[Y\right]$.

(b) Find $\operatorname{Var} Y$.

Problem 5 (Yates and Goodman, 2005, Q3.3.6). The cdf of random variable V is

$$F_V(v) = \begin{cases} 0 & v < -5, \\ (v+5)^2/144, & -5 \le v < 7, \\ 1 & v \ge 7. \end{cases}$$

(a) What is $f_V(v)$?

(b) What is $\mathbb{E}[V]$?

(c) What is $\mathrm{Var}[V]$?

(d) What is $\mathbb{E}[V^3]$?