ECS 315: In-Class Exercise # 9 - Sol

Date: 17 / 09 / 2019			
Name	ID (last 3 digits)		

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

(1) Suppose $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{6}$.

Find $P(A\cap B)$ to make events A and B independent.

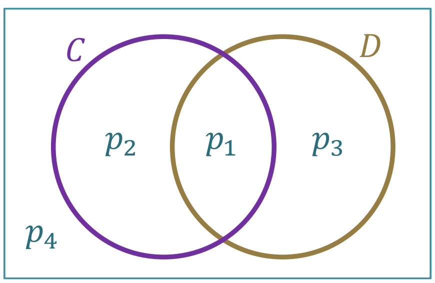
By definition, events A and B are independent if and only if $P(A\cap B)=P(A)P(B)$.

Therefore, $P(A\cap B)=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$.

(2) Suppose $P(C)=\frac{1}{4}$ and $P(C\cup D)=\frac{5}{8}$.

Find $P(C\cap D)$ to make events C and D independent.

We will use a systematic approach. Consider the Venn diagram below.



As usual, for two events we partition the sample space (Ω) into 4 parts.

Let p_i be the probability of the i^{th} part.

Observe that $P(C\cap D)$ is p_1.

From the provided information, we know that

$$p_1+p_2=P(C)=\frac{1}{4}. \quad (1)$$

$$p_1+p_2+p_3=P(C\cup D)=\frac{5}{8}. \quad (2)$$

As usual, we also know that

$$p_1+p_2+p_3+p_4=1. \quad (3)$$

The requirement that events C and D must be independent means

$$P(C\cap D)=P(C)P(D)$$

This is equivalent to

$$p_1=(p_1+p_2)(p_2+p_3). \quad (4)$$

Note that we now have four equations to solve for four unknowns. This should be possible to do. Here, it requires only a few more steps to solve for p_1.

$$p_1=\frac{1}{4}\left(\frac{5}{8}-p_1\right).$$

$$p_1=\frac{1}{8}.$$

There are usually other “easier” solutions. However, they are not as systematic as the solution above.

For example, one can start with

$$P(C \cup D) = P(C) + P(D) - P(C \cap D).$$

Forcing the events C and D to be independent means we must have $P(C \cap D) = P(C)P(D)$.

So,

$$P(C \cup D) = P(C) + P(D) - P(C)P(D).$$

Plugging in the provided values, we have

$$\frac{5}{8} = \frac{1}{4} + P(D) - \frac{1}{4}P(D).$$

This gives

$$P(D) = \frac{1}{2}.$$

Therefore,

$$P(C \cap D) = P(C)P(D) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}.$$