ECS 315: In-Class Exercise # 8 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

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$$P(\text{HIV}) = \frac{1}{25}$$

Suppose that for Westeros, 1 in 25 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (-) response. Suppose the test gives the correct answer 75% of the time. (The test is 75% accurate.)

This part of the problem gives two conditional probabilities:

$$P(+|\text{HIV}) = P(-|\text{HIV}^c) = 0.75 = \frac{3}{4}$$

(a) What is P(−|HIV), the conditional probability that a randomly-chosen person tests negative given that the person does have the HIV virus?

Recall that $P(A^c|B) = 1 - P(A|B)$.

Therefore, $P(-|\text{HIV}) = 1 - P(+|\text{HIV}) = 1 - \frac{3}{4} = \frac{1}{4}$.

(b) Find the probability that a randomly-chosen person tests positive.

By the total probability theorem,

$$P(+) = P(+|\text{HIV})P(\text{HIV}) + P(+|\text{HIV}^c)P(\text{HIV}^c)$$
$$= \frac{3}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{24}{25} = \frac{27}{100} = 0.27.$$

$$P(+|\text{HIV}^c) = 1 - P(-|\text{HIV}^c) = 1 - \frac{3}{4} = \frac{1}{4}.$$

(c) Find the conditional probability that a randomly-chosen person has the HIV virus given that the person tests positive.

By "Form 1" of the Bayes' theorem,

$$P(\text{HIV}|+) = \frac{P(+|\text{HIV})P(\text{HIV})}{P(+)} = \frac{\frac{3}{4} \times \frac{1}{25}}{\frac{27}{100}} = \frac{3}{27} = \frac{1}{9} \approx 0.11.$$