ECS 315: In-Class Exercise # 4

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 29/ 08/ 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

In each of the parts below, find $P(A)$, $P(B)$, and $P(A \cap B)$.

(a) $P(A^c) = 0.5$, $P(A \cup B) = 0.6$, and $P(B^c) = 0.7$.

$P(A) = 1 - P(A^c) = 1 - 0.5 = 0.5$

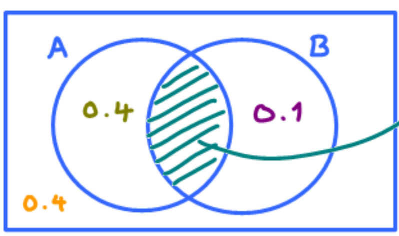
$P(B) = 1 - P(B^c) = 1 - 0.7 = 0.3$

From (5.16), $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Therefore, $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.3 - 0.6 = 0.2$

$P(A) =$ 0.5, $P(B) =$ 0.3, and $P(A \cap B) =$ 0.2.

(b) $P(A^c \cap B^c) = 0.4$, $P(A \cap B^c) = 0.4$, and $P(A^c \cap B) = 0.1$.



We know that $P(\Omega) = 1$

Here, we must have

$0.4 + 0.4 + P(A \cap B) + 0.1 = 1.$

Therefore, $P(A \cap B) = 0.1$

$P(A) = P(A \cap B^c) + P(A \cap B)$
$= 0.4 + 0.1$
$= 0.5$

$P(B) = P(A \cap B) + P(A^c \cap B)$
$= 0.1 + 0.1$
$= 0.2$

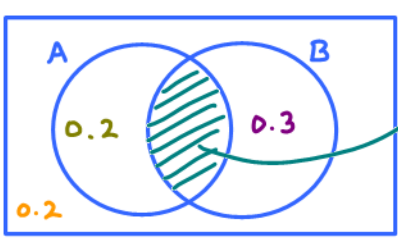
$P(A) =$ 0.5, $P(B) =$ 0.2, and $P(A \cap B) =$ 0.1.

(c) $P(A \cup B^c) = 0.7$, $P(A^c \cup B) = 0.8$, $P(A \cup B) = 0.8$.

$P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

$P(A^c \cap B) = 1 - P(A \cup B^c) = 1 - 0.7 = 0.3$

$P(A \cap B^c) = 1 - P(A^c \cup B) = 1 - 0.8 = 0.2$



We know that $P(\Omega) = 1$

Here, we must have

$0.2 + 0.2 + P(A \cap B) + 0.3 = 1.$

Therefore, $P(A \cap B) = 0.3$

$P(A) = P(A \cap B^c) + P(A \cap B)$
$= 0.2 + 0.3$
$= 0.5$

$P(B) = P(A \cap B) + P(A^c \cap B)$
$= 0.3 + 0.3$
$= 0.6$

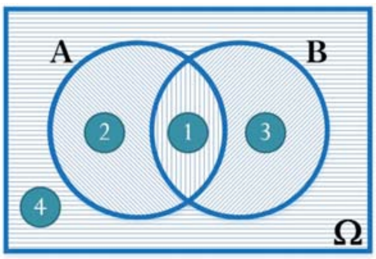
$P(A) =$ 0.5, $P(B) =$ 0.6, and $P(A \cap B) =$ 0.3.

Tips for Finding Event-Based Probability

- Don't forget that we always have an extra piece of information: $P(\Omega) = 1$.
- It is easier to work with expression involving intersection than the one with union.
  - Use de Morgan law [2.5] and complement rule [5.15]
  - For example, suppose we are given that $P(A \cup B^c) = 0.3$.
    - By the complement rule, $P\left((A \cup B^c)^c\right) = 1 - 0.3 = 0.7$.
    - By de Morgan law, $(A \cup B^c)^c = A^c \cap B$.
    - Therefore, the provided information is equivalent to $P(A^c \cap B) = 0.7$.

Tips for Finding Event-Based Probability

- Given n events, the sample space (Ω) can be partitioned into 2^n parts where each part is an intersection of the events or their complements.
- For example, when we have two events, the sample space can be partitioned into 4 parts:
  ❶ $A \cap B$,
  ❷ $A \cap B^c$,
  ❸ $A^c \cap B$, and
  ❹ $A^c \cap B^c$
  as shown in the Venn diagram.



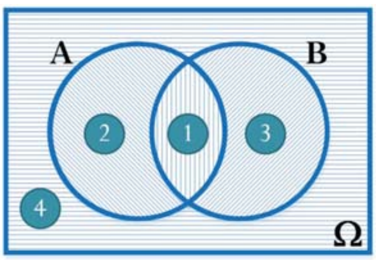
- Any event can be written as a disjoint union of these parts. Therefore, if we can find the probabilities for these parts, then we can find the probability for any event by adding the probabilities of the corresponding parts.

Tips for Finding Event-Based Probability

- If your aim is simply to find one working method to solve a problem (not trying to find the smart way to solve it), then the steps on the next slide will be helpful.
- It turns the problem into solving system of linear equations.

Steps to Find Event-Based Probability

- Let n be the number of events' names used in the question.
  - For example, if the question only talks about A and B, then $n = 2$.
- Partition the sample space (Ω) into 2^n parts where each part is an intersection of the events or their complements.
  - For example, when we have two events, the sample space can be partitioned into 4 parts:
    ❶ $A \cap B$,
    ❷ $A \cap B^c$,
    ❸ $A^c \cap B$, and
    ❹ $A^c \cap B^c$
    as shown in the Venn diagram.



- Let p_i be the probability of the i^{th} part.

Steps to Find Event-Based Probability

- Turn the given information into equation(s) of the p_i.
  - For example, if you are given that $P(A \cup B) = 0.3$, we see that $A \cup B$ cover parts ❶, ❷, and ❸. Therefore, by finite additivity, the corresponding equation is $p_1 + p_2 + p_3 = 0.3$.
  - It is easier to work with expression involving intersection than the one with union.
    - Use de Morgan law [2.5] and complement rule [5.15]
    - For example, suppose we are given that $P(A \cup B^c) = 0.3$.
      - By the complement rule, $P\left((A \cup B^c)^c\right) = 1 - 0.3 = 0.7$.
      - By de Morgan law, $(A \cup B^c)^c = A^c \cap B$.
      - Therefore, the provided information is equivalent to $P(A^c \cap B) = 0.7$.
      - The corresponding equation is $p_3 = 0.7$.
  - Don't forget that we always have an extra piece of information: $P(\Omega) = 1$.
    - With two events, this means $p_1 + p_2 + p_3 + p_4 = 1$.

Steps to Find Event-Based Probability

- Solve for the values of the p_i.
  - Note that there are n unknowns; so we will need n equations to solve for the values of the p_i.
  - If we don't have enough equations, you may be overlooking some given piece(s) of information or it is possible that you don't need to know the values of all the p_i to find the final answer(s).
- The probability of any event can be found by adding the probabilities of the corresponding parts.

Here, we follow "Steps to Find Event-Based Probability" in the slides.

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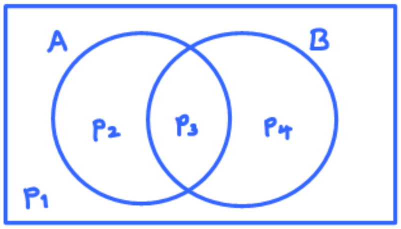
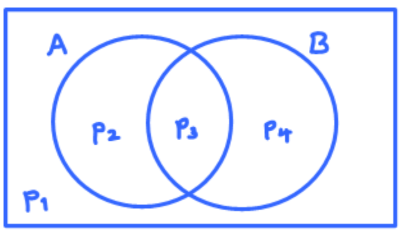


$$\left.\begin{array}{l} p_1 + p_4 = 0.5 \\ p_1 + p_2 = 0.7 \\ p_2 + p_3 + p_4 = 0.6 \\ p_1 + p_2 + p_3 + p_4 = 1 \end{array}\right\} \Rightarrow \begin{array}{l} p_1 = 0.4 \\ p_2 = 0.3 \\ p_3 = 0.2 \\ p_4 = 0.1 \end{array}$$

$P(A) = \underset{p_2 + p_3}{\underline{0.5}}$, $P(B) = \underset{p_3 + p_4}{\underline{0.3}}$, and $P(A \cap B) = \underset{p_3}{\underline{0.2}}$.

(b) $P(A^c \cap B^c) = 0.4$, $P(A \cap B^c) = 0.4$, and $P(A^c \cap B) = 0.1$.



$p_1 = 0.4$

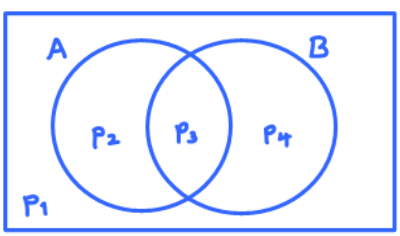
$p_2 = 0.4$

$p_4 = 0.1$

$p_1 + p_2 + p_3 + p_4 = 1 \Rightarrow p_3 = 1 - 0.4 - 0.4 - 0.1 = 0.1$

$P(A) = \overset{p_2 + p_3}{\underline{0.5}}$, $P(B) = \overset{p_3 + p_4}{\underline{0.2}}$, and $P(A \cap B) = \overset{p_3}{\underline{0.1}}$.

(c) $P(A \cup B^c) = 0.7$, $P(A^c \cup B) = 0.8$, $P(A \cup B) = 0.8$.



$$\left.\begin{array}{l} p_2 + p_3 + p_4 = 0.8 \\ p_1 + p_2 + p_3 = 0.7 \\ p_1 + p_3 + p_4 = 0.8 \\ p_1 + p_2 + p_3 + p_4 = 1 \end{array}\right\} \begin{array}{l} p_1 = 0.2 \\ p_2 = 0.2 \\ p_3 = 0.3 \\ p_4 = 0.3 \end{array}$$

$P(A) = \overset{p_2 + p_3}{\underline{0.5}}$, $P(B) = \overset{p_3 + p_4}{\underline{0.6}}$, and $P(A \cap B) = \overset{p_3}{\underline{0.3}}$.