ECS 315: In-Class Exercise # 2 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as your group** for the previous exercise.
2. [ENRE] = Explanation is not required for this exercise.
3. **Do not panic.**

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[ENRE] Let

$\mathbb{N} =$ the set of all natural numbers,

$A =$ the interval $[0,2]$,

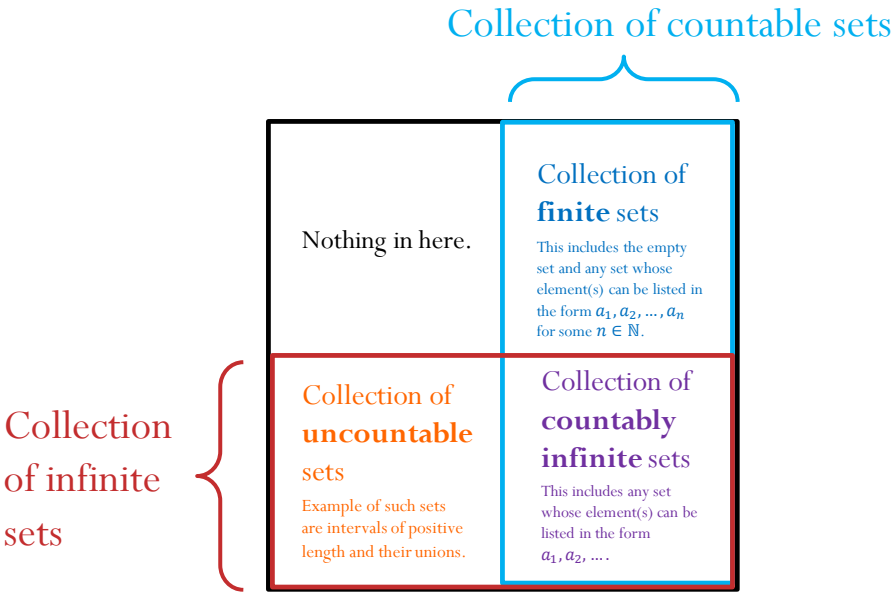
$B =$ the set of all real-valued x satisfying $\cos(x) = x^2 + 2$, and

$C =$ the set of all real-valued x satisfying $\cos(x) \geq 0.5$.

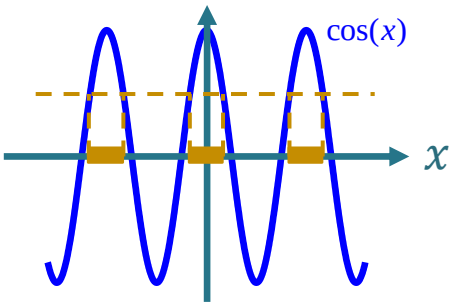
For each of the sets provided in the first column of the table below, indicate (by putting a Y(es) or an N(o) in each corresponding cell) whether it is "finite", "infinite", "countably infinite", "uncountable".

	Finite	Infinite	Countably Infinite	Uncountable
$\mathbb{N}$	N	Y	Y	N
$\{1,2,\ldots,10^{10}\}$	Y	N	N	N
A	N	Y	N	Y
B	Y	N	N	N
C	N	Y	N	Y

First, we find the "key" type of each given set. (Figure 4 from the lecture notes is copied below.)



- $\mathbb{N}=\{1,2,3,\ldots\}$ is countably infinite. We used it as the first example of countable infinite sets. In particular, its members can be listed in the form $a_1,a_2,a_3,\ldots$ by setting $a_k=k$.
- Although it has a lot of members, the set $\{1,2,\ldots,10^{10}\}$ is still finite. Its size is 10^{10} which is large but not ∞.
- Any interval with positive length is an uncountable set. Therefore, A is uncountable.
- Note that $\cos(x)\le 1$ but $x^2+2\ge 2$. Therefore, the two functions will never intersect. Therefore, $B=\varnothing$ which is finite.
- For set C, one can try to make a lousy plot of $\cos(x)$ and locate the x values that give $\cos(x)\ge 0.5$. This is shown below:



Observe that these x values correspond to a union of intervals all of which have positive length. Therefore, C is uncountable.

	Finite	Infinite	Countably Infinite	Uncountable
$\mathbb{N}$			Y	
$\{1,2,\ldots,10^{10}\}$	Y			
A				Y
B	Y			
C				Y

Then, we can apply the following reasoning:

- Any countably infinite set is, by definition, infinite and hence not finite.
  Furthermore, any countably infinite set is, by definition, countable and hence not uncountable.
  So, the answers for the corresponding row are N Y Y N.
- Any finite set cannot be infinite, countably infinite, nor uncountable.
  So, the answers for the corresponding row are Y N N N.
- Any uncountable set is infinite. Any infinite set is not finite.
  Furthermore, any uncountable set is, by definition, not countable and therefore cannot be countably infinite.
  So, the answers for the corresponding row are N Y N Y.