ECS 315: In-Class Exercise # 25 - Sol

Instructions

1. Separate into groups of no more than three students each.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 2 5 / 1 1 / 2019			
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1. Random variables X and Y have the following joint pmf matrix

$$\mathbf{P}_{X,Y} = \begin{array}{c} x \backslash y \\ 1 \\ 0 \end{array} \begin{array}{c} 0 \quad 1 \\ \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & c \end{bmatrix} \end{array}$$

Are X and Y uncorrelated?

$c = 1 - (0.1 + 0.2 + 0.3) = 1 - 0.6 = 0.4.$

$$\mathbb{E}[XY] = 0 \times 1 \times 0.1 + 0 \times 0 \times 0.2 + 1 \times 1 \times 0.3 + 0 \times 1 \times 0.4 = 0.3$$

$\mathbf{P}_{X,Y}$: rows $x = 1$: $0.1, 0.3$; $x = 0$: $0.2, 0.4$. Row sums: 0.4, 0.6. Column sums: 0.3, 0.7.

$\mathbb{E}X = 1 \times 0.4 + 0 \times 0.6 = 0.4$

$\mathbb{E}Y = 0 \times 0.3 + 1 \times 0.7 = 0.7$

$(\mathbb{E}X)(\mathbb{E}Y) = 0.28$

$(\mathbb{E}X)(\mathbb{E}Y) \neq \mathbb{E}[XY]$ ➔ $\mathrm{Cov}[X,Y] \neq 0$ ➔ No, X and Y are not uncorrelated.

2. Random variables X and Y have the following joint pmf matrix

$$\mathbf{P}_{X,Y} = \begin{array}{c} x \backslash y \\ 1 \\ 0 \end{array} \begin{array}{c} 0 \quad 1 \\ \begin{bmatrix} 0.1 & a \\ 0.2 & b \end{bmatrix} \end{array}$$

Suppose X and Y are uncorrelated. Find the values of the unknown constants:

$a = \underline{\frac{7}{30} \approx 0.2333}$, $b = \underline{\frac{7}{15} \approx 0.4667}$.

$\mathbf{P}_{X,Y}$: rows $x = 1$: $0.1, a$; $x = 0$: $0.2, b$. Row sums: $0.1 + a$, $0.2 + b$. Column sums: 0.3, $a + b = 1 - 0.3 = 0.7$.

$$\mathbb{E}[XY] = (0)(1)(0.1) + (0)(0)(0.2) + (1)(1)(a) + (0)(1)(b) = a$$

$\mathbb{E}X = 1 \times (0.1 + a) + 0 \times (0.2 + b) = 0.1 + a$

$\mathbb{E}Y = 0 \times (0.3) + 1 \times (0.7) = 0.7$

For X and Y to be uncorrelated, we need

$\mathbb{E}[XY] = (\mathbb{E}X)(\mathbb{E}Y)$

$a = (0.1 + a)(0.7)$

$a = \frac{7}{30} \approx 0.2333.$

$b = 0.7 - \frac{7}{30} = \frac{7}{15} \approx 0.4667.$