

ECS 315: In-Class Exercise # 24 - Sol

Instructions

1. Separate into groups of no more than three students each.
2. [ENRE] Explanation is not required for this exercise.
3. **Do not panic.**

Date: <u>21</u> / <u>11</u> / <u>2019</u>			
Name			ID (last 3 digits)

1) Suppose the pmf of a random variable X is given by

$$p_X(x) = \begin{cases} 0.1, & x = 2, \\ c, & x = 4, \\ 0, & \text{otherwise.} \end{cases}$$

Let Y be another random variable. Assume that X and Y are i.i.d.

Find

a) $c = \underline{0.9}$

pmf \Rightarrow " $\Sigma = 1$ " $\Rightarrow 0.1 + c = 1 \Rightarrow c = 0.9 \Rightarrow p_X(x) = \begin{cases} 0.1, & x = 2, \\ 0.9, & x = 4, \\ 0, & \text{otherwise.} \end{cases}$

X and Y are identically distributed.

b) Their joint pmf matrix $\mathbf{P}_{X,Y}$.

X and Y are independent.

$$p_{X,Y}(x,y) = p_X(x)p_Y(y)$$

$$\mathbf{P}_{X,Y} = \begin{matrix} & x \backslash y & 2 & 4 \\ \begin{matrix} 2 \\ 4 \end{matrix} & \begin{bmatrix} p_X(2)p_Y(2) & p_X(2)p_Y(4) \\ p_X(4)p_Y(2) & p_X(4)p_Y(4) \end{bmatrix} & = & \begin{bmatrix} (0.1)(0.1) & (0.1)(0.9) \\ (0.9)(0.1) & (0.9)(0.9) \end{bmatrix} & = & \begin{matrix} x \backslash y & 2 & 4 \\ 2 & \begin{bmatrix} 0.01 & 0.09 \\ 0.09 & 0.81 \end{bmatrix} \\ 4 & \end{matrix} \end{matrix}$$

2) Random variables X and Y are independent. Their joint pmf matrix is

$$\mathbf{P}_{X,Y} = \begin{matrix} & x \backslash y & 2 & 5 & 8 \\ \begin{matrix} -1 \\ 3 \end{matrix} & \begin{bmatrix} 0.08 & 0.12 & a \\ 0.12 & b & c \end{bmatrix} \end{matrix}$$

Find the values of the unknown constants:

$$a = \underline{0.2}, \quad b = \underline{0.18}, \quad c = \underline{0.3}$$

$$p_{X,Y}(-1,2) = p_X(-1)p_Y(2)$$

$$0.08 = (0.08 + 0.12 + a)(0.08 + 0.12)$$

$$a = 0.2$$

$$"\Sigma = 1"$$

$$1 = 0.08 + 0.12 + 0.2 + 0.12 + 0.18 + c$$

$$c = 0.3$$

$$p_{X,Y}(-1,5) = p_X(-1)p_Y(5)$$

$$0.12 = (0.08 + 0.12 + 0.2)(0.12 + b)$$

$$b = 0.18$$