ECS 315: In-Class Exercise # 23 - Sol

Instructions

	Name	ID (last 3 digits)		
	Prapun	5	5	5

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Random variables X and Y have the following joint pmf

$$p_{X,Y}(x, y) = \begin{cases} c(x+y)^2, & x \in \{-1, 2\} \text{ and } y \in \{0, -3\}, \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c

$$x+y \quad \begin{array}{c|cc} x \backslash y & 0 & -3 \\ \hline -1 & -1 & -4 \\ 2 & 2 & -1 \end{array}$$

$$c(x+y)^2 \quad \begin{array}{c|cc} x \backslash y & 0 & -3 \\ \hline -1 & c & 16c \\ 2 & 4c & c \end{array}$$

$$\sum_x \sum_y p_{X,Y}(x,y) = c + 16c + 4c + c = 22c$$

$$1 = 22c$$

$$c = \frac{1}{22}$$

b) Find the joint pmf matrix $\mathbf{P}_{X,Y}$

$$p_{X,Y}(x, y) = c(x+y)^2 \quad \begin{array}{c|cc} x \backslash y & 0 & -3 \\ \hline -1 & c & 16c \\ 2 & 4c & c \end{array}$$

$$\mathbf{P}_{X,Y} \quad \begin{array}{c|cc} x \backslash y & 0 & -3 \\ \hline -1 & \frac{1}{22} & \frac{16}{22} \\ 2 & \frac{4}{22} & \frac{1}{22} \end{array} = \begin{array}{c|cc} x \backslash y & 0 & -3 \\ \hline -1 & \frac{1}{22} & \frac{8}{11} \\ 2 & \frac{2}{11} & \frac{1}{22} \end{array}$$

c) Find $P[X > Y]$.

We circle only the elements whose $x > y$.

$$\mathbf{P}_{X,Y} \quad \begin{array}{c|cc} x \backslash y & 0 & -3 \\ \hline -1 & \frac{1}{22} & \boxed{\frac{16}{22}} \\ 2 & \boxed{\frac{4}{22}} & \boxed{\frac{1}{22}} \end{array}$$

$$P[X > Y] = \frac{16 + 4 + 1}{22} = \frac{21}{22} \approx 0.9545$$

d) Find the pmf $p_X(x)$ and the pmf $p_Y(y)$.

$$\mathbf{P}_{X,Y} \quad \begin{array}{c|cc} x \backslash y & 0 & -3 \\ \hline -1 & \frac{1}{22} & \frac{16}{22} \\ 2 & \frac{4}{22} & \frac{1}{22} \end{array} \quad \begin{array}{c} \xrightarrow{\Sigma} \frac{17}{22} \\ \xrightarrow{\Sigma} \frac{5}{22} \end{array}$$

$$\downarrow \Sigma \quad \downarrow \Sigma$$

$$\frac{5}{22} \quad \frac{17}{22}$$

$$p_Y(y) = \begin{cases} \frac{5}{22}, & y = 0, \\ \frac{17}{22}, & y = -3, \\ 0, & \text{otherwise} \end{cases}$$

$$p_X(x) = \begin{cases} \frac{17}{22}, & x = -1, \\ \frac{5}{22}, & x = 2, \\ 0, & \text{otherwise} \end{cases}$$