# ECS 315: In-Class Exercise # 22 - Sol

## Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Calculate $P[0.5 < X \le 1.5]$ for each of the following random variables.

Your answer should be of the form 0.XXXX.

a) $X \sim \mathcal{U}(\overset{a}{1}, \overset{b}{4})$

$$f_X(X) = \begin{cases} \dfrac{1}{b-a}, & a < x < b, \\ 0, & \text{otherwise.} \end{cases}$$

$$= \begin{cases} \dfrac{1}{3}, & 1 < x < 4, \\ 0, & \text{otherwise.} \end{cases}$$

$$P[0.5 < X < 1.5] = \int_{0.5}^{1.5} f_X(x)\,dx$$

$$= \int_{0.5}^{1} 0\,dx + \int_{1}^{1.5} \frac{1}{3}\,dx$$

$$= 0 + \frac{1}{3}x\Big|_{x=1}^{1.5} = \frac{1}{6}$$

$$\approx 0.1667$$

b) $X \sim \mathcal{E}(\overset{\lambda}{1})$

$$f_X(X) = \begin{cases} \lambda e^{-\lambda x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$= \begin{cases} e^{-x}, & x > 0, \\ 0, & \text{otherwise.} \end{cases}$$

$$P[0.5 < X < 1.5] = \int_{0.5}^{1.5} f_X(x)\,dx$$

$$= \int_{0.5}^{1.5} e^{-x}\,dx = -e^{-x}\big|_{x=0.5}^{1.5}$$

$$= e^{-0.5} - e^{-1.5}$$

$$\approx 0.6065 - 0.2231$$

$$= 0.3834$$

c) $X \sim \mathcal{N}(\overset{m,\,\sigma^2}{0,1})$

$$P[0.5 < X < 1.5] = F_X(1.5) - F_X(0.5) = \Phi(1.5) - \Phi(0.5)$$

$$\approx 0.9332 - 0.6915 = 0.2417$$

We can use the $\phi$-function directly here because $X$ is a standard Gaussian RV.

d) $X \sim \mathcal{N}(\overset{m,\,\sigma^2}{1,3})$

$$P[0.5 < X < 1.5] = F_X(1.5) - F_X(0.5) = \Phi\left(\frac{1.5-1}{\sqrt{3}}\right) - \Phi\left(\frac{0.5-1}{\sqrt{3}}\right)$$

$$\approx \Phi(0.29) - \Phi(-0.29) = \Phi(0.29) - \big(1 - \Phi(0.29)\big) \approx 2\Phi(0.29) - 1$$

$$= 2 \times 0.6141 - 1 = 0.2282$$