

ECS 315: In-Class Exercise # 21

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: <u>12</u> / <u>11</u> / 2019			
Name			ID <small>(last 3 digits)</small>
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In this question, we consider two distributions for a random variable X .

In part (a), which corresponds to the second column in the table below, X is a **discrete** random variable with its pmf specified in the first row.

In part (b), which corresponds to the third column, X is a **continuous** random variable with its pdf specified in the first row.

	$p_X(x) = \begin{cases} \frac{1}{5}x^2, & x \in \{-1, 2\}, \\ 0, & \text{otherwise.} \end{cases}$	$f_X(x) = \begin{cases} \frac{1}{3}x^2, & x \in (-1, 2], \\ 0, & \text{otherwise.} \end{cases}$
Find $\mathbb{E}X$	$\begin{aligned} \mathbb{E}X &= \sum_x x p_X(x) = (-1) \left(\frac{1}{5}\right) + (2) \left(\frac{4}{5}\right) \\ &= \frac{7}{5} = 1.4 \end{aligned}$	$\begin{aligned} \mathbb{E}X &= \int_{-\infty}^{\infty} x f_X(x) dx = \int_{-1}^2 x \left(\frac{1}{3}x^2\right) dx \\ &= \frac{1}{3} \frac{x^4}{4} \Big _{-1}^2 = \frac{1}{12} (16 - 1) = \frac{15}{12} = \frac{5}{4} \\ &= 1.25 \end{aligned}$
Find $\mathbb{E}[X^2]$	$\begin{aligned} \mathbb{E}[X^2] &= \sum_x x^2 p_X(x) \\ &= (-1)^2 \left(\frac{1}{5}\right) + (2)^2 \left(\frac{4}{5}\right) \\ &= \frac{17}{5} = 3.4 \end{aligned}$	$\begin{aligned} \mathbb{E}[X^2] &= \int_{-\infty}^{\infty} x^2 f_X(x) dx \\ &= \int_{-1}^2 x^2 \left(\frac{1}{3}x^2\right) dx = \frac{1}{3} \int_{-1}^2 x^4 dx \\ &= \frac{1}{3} \frac{x^5}{5} \Big _{-1}^2 = \frac{1}{15} (32 - (-1)) = \frac{33}{15} \\ &= \frac{11}{5} = 2.2 \end{aligned}$
Find $\text{Var}[X]$	$\begin{aligned} \text{Var}[X] &= \mathbb{E}[X^2] - (\mathbb{E}X)^2 \\ &= \frac{17}{5} - \left(\frac{7}{5}\right)^2 = \frac{85 - 49}{25} \\ &= \frac{36}{25} = 1.44 \end{aligned}$	$\begin{aligned} \text{Var}[X] &= \mathbb{E}[X^2] - (\mathbb{E}X)^2 \\ &= \frac{11}{5} - \left(\frac{5}{4}\right)^2 = \frac{176 - 125}{80} = \frac{51}{80} \\ &\approx 0.6375 \end{aligned}$
Find σ_X	$\sigma_X = \sqrt{\text{Var}[X]} = \sqrt{\frac{36}{25}} = \frac{6}{5} = 1.2$	$\sigma_X = \sqrt{\text{Var}[X]} = \sqrt{\frac{51}{80}} = \frac{1}{4} \sqrt{\frac{51}{5}} \approx 0.7984$