

ECS 315: In-Class Exercise # 20

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. [ENRE] Explanation is not required for this exercise..
3. **Do not panic.**

Date: <u>07/11/2019</u>			
Name			ID (last 3 digits)
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In this question, we consider two distributions for a random variable X .

In part (a), which corresponds to the second column in the table below, X is a **discrete** random variable with its pmf specified in the first row.

In part (b), which corresponds to the third column, X is a **continuous** random variable with its pdf specified in the first row.

	$p_X(x) = \begin{cases} \frac{1}{5}x^2, & x \in \{-1, 2\}, \\ 0, & \text{otherwise.} \end{cases}$	$f_X(x) = \begin{cases} \frac{1}{3}x^2, & x \in (-1, 2], \\ 0, & \text{otherwise.} \end{cases}$
Find the cdf $F_X(x)$	<p style="color: orange;">For discrete RV, it may be easier to work on the plot of cdf first, then come back here.</p> $F_X(x) = \begin{cases} 0, & x < -1, \\ 0.2, & -1 \leq x < 2, \\ 1 & x \geq 2. \end{cases}$ <p style="color: orange;">Note that, for discrete RVs, their cdf's are not continuous. At all points, including the jumps, the cdf's must be right-continuous. Therefore, here, the interval that $F_X(x) = 0.2$ must include $x = -1$ and exclude $x = 2$.</p>	$F_X(x) = \int_{-\infty}^x f_X(t) dt$ <p>For $x \leq -1$, $F_X(x) = \int_{-\infty}^x 0 dt = 0$.</p> <p>For $-1 < x \leq 2$,</p> $F_X(x) = \int_{-\infty}^{-1} 0 dt + \int_{-1}^x \frac{1}{3}t^2 dt$ $= 0 + \frac{t^3}{9} \Big _{-1}^x = \frac{x^3 + 1}{9}.$ <p>For $x > 2$, $F_X(x) = \int_{-\infty}^2 \frac{1}{3}t^2 dt + \int_2^{\infty} 0 dt = 1$.</p> <p>Therefore,</p> $F_X(x) = \begin{cases} 0, & x \leq -1, \\ \frac{x^3 + 1}{9}, & -1 < x \leq 2, \\ 1 & x > 2. \end{cases}$
Plot the cdf $F_X(x)$	<p>To get the cdf from the pmf, locate where the probability masses are from the pmf. Then, start from $-\infty$ with the value 0. Moving to the right, we increase the cdf value at the locations of the probability masses. The amount of jump is the same as the amount of probability mass at that location.</p>	<p>The expression in the previous part gives</p>