ECS 315: In-Class Exercise # 17 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 2 9 / 1 0 / 2019

Name	ID (last 3 digits)		
Prapun	5	5	5

1. Find the expected value of the random variable X defined in each part below:

a. $p_X(x) = \begin{cases} cx, & x \in \{1,2\}, \\ 0, & \text{otherwise.} \end{cases}$

First, we need to solve for the value of the unknown constant c.
To be a pmf, we need "$\Sigma = 1$". So,

$$p_X(1) + p_X(2) = 1$$
$$c + 2c = 1$$
$$c = \frac{1}{3}.$$

x	$p_X(x)$
1	$c = \frac{1}{3}$
2	$2c = 2 \times \frac{1}{3} = \frac{2}{3}$

$$\mathbb{E}X = \sum_x x p_X(x) = \left(1 \times p_X(1)\right) + \left(2 \times p_X(2)\right)$$
$$= \left(1 \times \frac{1}{3}\right) + \left(2 \times \frac{2}{3}\right) = \frac{5}{3} \approx 1.67.$$

b. $p_X(x) = \begin{cases} 0.3, & x = -1, 1, \\ c, & x = 3, \\ 0, & \text{otherwise.} \end{cases}$

"$\Sigma = 1$": $\quad p_X(-1) + p_X(1) + p_X(3) = 1$
$$0.3 + 0.3 + c = 1$$
$$c = 0.4.$$

x	$p_X(x)$
-1	0.3
1	0.3
3	$c = 0.4$

$$\mathbb{E}X = \sum_x x p_X(x) = (-1 \times 0.3) + (1 \times 0.3) + (3 \times 0.4) = 1.2.$$

c. $F_X(x) = \begin{cases} 0, & x < -1, \\ 0.4, & -1 \le x < 1, \\ 1, & x \ge 1. \end{cases}$

This cdf has two jumps; one is @ $x = -1$ and another one is @ $x = 1$.

The jump sizes are 0.4 and 0.6, respectively.

x	$p_X(x)$
-1	0.4
1	0.6

$$\mathbb{E}X = \sum_x x p_X(x) = (-1 \times 0.4) + (1 \times 0.6) = 0.2.$$

d. $p_X(x) = \begin{cases} cx, & x \in \{1,2,3,\ldots,10\}, \\ 0, & \text{otherwise.} \end{cases}$

"$\Sigma = 1$": $\sum_{x=1}^{10} cx = 1 \Rightarrow c = \frac{1}{\sum_{x=1}^{10} x} = \frac{2}{10 \times 11}$.

$$\mathbb{E}X = \sum_x x p_X(x) = \sum_{x=1}^{10} x(cx) = c\sum_{x=1}^{10} x^2 = c\left(\frac{1}{6} \times 10 \times 11 \times 21\right) = 7.$$

$$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2} = \frac{n^2 + n}{2}$$
$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6} = \frac{2n^3 + 3n^2 + n}{6}$$
$$1^3 + 2^3 + 3^3 + \cdots + n^3 = \left[\frac{n(n+1)}{2}\right]^2 = \frac{n^4 + 2n^3 + n^2}{4}$$