ECS 315: In-Class Exercise # 16 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 24/10/2019			
Name	ID (last 3 digits)		
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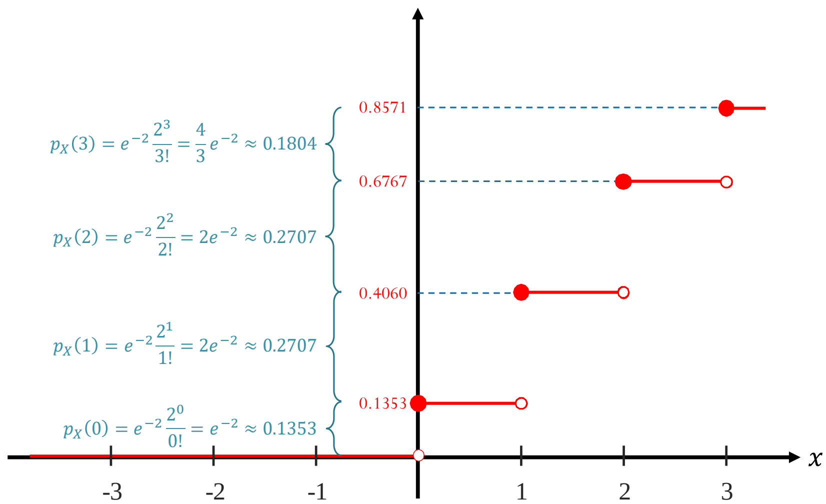
1. Arrivals of customers at a restaurant are modeled by a Poisson process with a rate of $\lambda = \frac{1}{3}$ customers per minute. Let X be the number of customers arriving between 5:00 PM and 5:06 PM.

   a. Plot its cdf $F_X(x)$ on the interval $[-3,3]$.



We know that $X \sim \mathcal{P}(\alpha)$ where $\alpha = \lambda T$. Here, $\lambda = \frac{1}{3}$ [customers per minute] and $T = 6$ [minutes]. Therefore,

$$\alpha = \frac{1}{3} \times 6 = 2.$$

Being a Poisson RV, we also know that

$$p_X(x) = \begin{cases} e^{-\alpha}\frac{\alpha^x}{x!}, & x = 0,1,2,\ldots, \\ 0, & \text{otherwise.} \end{cases}$$

   b. What is the probability that $X < 1$?

   Because $X \sim \mathcal{P}(\alpha)$, the possible values of X are 0,1,2,3, … Among its possible values, the only value that is "< 1" is "0". Therefore,

$$P[X < 1] = P[X = 0] = p_X(0) = e^{-2}\frac{2^0}{0!} = e^{-2} \approx 0.1353.$$

2. Let N be the number of successes in 10^{20} Bernoulli trials. Assume that the probability of success for each trial is 10^{-21}. Use Poisson approximation to calculate $P[N = 0]$.

Your answer should be of the form 0.XXXX.

First, note that $N \sim \mathcal{B}(n,p)$ where $n = 10^{20}$ and $p = 10^{-21}$. Because n is large and p is small, we can approximate the pmf of N by that of the Poisson pmf whose $\alpha = np$. Here, $np = 10^{20} \times 10^{-21} = \frac{1}{10}$. Therefore,

$$P[N = n] \approx \frac{e^{-\alpha}\alpha^n}{n!} \text{ for } n = 0,1,2,\ldots$$

Substituting $\alpha = 10$, we have

$$P[N = 0] \approx \frac{e^{-\frac{1}{10}}10^0}{0!} = e^{-\frac{1}{10}} \approx 0.9048.$$