ECS 315: In-Class Exercise # 15 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

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Name	ID (last 3 digits)		
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1. Suppose $X \sim \mathcal{G}_0\left(\frac{1}{3}\right)$. Find $P\left[X \le \frac{1}{3}\right]$.

$\rightarrow p_X(x) = \begin{cases} p(1-p)^x, & x = 0,1,2,\dots, \\ 0, & \text{otherwise.} \end{cases}$

The possible values of X are $0,1,2,\dots$. Among these, only 0 satisfies the condition "$\le \frac{1}{3}$".

Therefore, $P\left[X \le \frac{1}{3}\right] = p_X(0) = p(1-p)^0 = p = \frac{1}{3}$.

2. Suppose $X \sim \mathcal{G}_1\left(\frac{1}{3}\right)$. Find $P\left[X \le \frac{1}{3}\right]$.

$\rightarrow p_X(x) = \begin{cases} p(1-p)^{x-1}, & x = 1,2,\dots, \\ 0, & \text{otherwise.} \end{cases}$

Note that the possible values (with positive probability) of X are always ≥ 1.

Therefore, $P\left[X \le \frac{1}{3}\right] = 0$.

3. [ENRPa] Consider (a sequence of independent) Bernoulli trials whose success probability for each trial is $1/4$. For each of the random variables defined below, indicate the **name and the parameter(s)** of the family it belongs to.

Random Variable	Family
$K =$ the number of failures until the first success occurs.	$\mathcal{G}_0\left(\frac{1}{4}\right)$
$N =$ the number of successes among the first 4 trials.	$\mathcal{B}\left(4, \frac{1}{4}\right)$
$F =$ the number of successes until the first failure occurs.	$\mathcal{G}_0\left(\frac{3}{4}\right)$

For K and N, see the interpretation part of Definitions 8.33 and 8.22. In addition, see Figure 22.

An easy way to think about F is to note that F and K are almost the same except that the roles of "success" and "failure" are switched. So, it makes sense that it still belongs to the same family with the parameter being replaced by the opposite case.

A formal way to solve this problem is to derive the pmf of F and compare the pmf to the known families. Here, because F is defined to be "the number of" something, we start by assuming that the possible values of F are $0,1,2,3,\dots$. $F = 0$ is possible because one may fail right at the first trial; this happens with probability $1 - \frac{1}{4} = \frac{3}{4}$. In general, for $k = 0,1,2,\dots$, $P[F = f] = \left(\frac{1}{4}\right)^f \times \frac{3}{4}$. By comparing the formula with the known families (Table 3 in the lecture notes), $P[F = f]$ fits the formula for the geometric$_0$ distribution; the corresponding p is $\frac{3}{4}$.