ECS 315: In-Class Exercise # 11 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Consider a random experiment in which you roll a six-sided fair dice (whose faces are numbered 1-6). We define the following random variables from the outcomes of this experiment:

$$X(\omega) = \omega \quad \text{and} \quad Y(\omega) = 2 + \big((\omega-1)(\omega-3)(\omega-5)(\omega-7)\big).$$

a. Find $P[X = 2]$.

$X(\omega) = 2$ when $\omega = 2$. Therefore, $P[X = 2] = P(\{2\}) = \frac{1}{6}$.

b. Find $P[Y = 2]$.

$Y(\omega) = 2$ when $2 + \big((\omega - 1)(\omega - 3)(\omega - 5)(\omega - 7)\big) = 2$.

Not in Ω.

$\omega = 1,3,5,\cancel{7}$

Therefore, $P[Y = 2] = P(\{1,3,5\}) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$.

2. Consider a random experiment in which you roll a 10-sided fair dice (whose faces are numbered 0–9). Define a random variable Z from the outcomes of this experiment by

$$Z(\omega) = (\omega - 6)^2.$$

$\Omega = \{0,1,2,\dots,9\}$

a. Find $P[Z = 4]$.

$Z(\omega) = 4$ when $(\omega - 6)^2 = 4$.

$\omega = 6 \pm 2 = 4$ or 8.

Therefore, $P[Z = 4] = P(\{4,8\}) = \frac{1}{10} + \frac{1}{10} = \frac{2}{10}$.

b. Find $P[Z > 20]$.

Method 1:

$Z(\omega) > 20$ when $(\omega - 6)^2 > 20$.

$\cancel{\omega > 6 + \sqrt{20}}$ or $\omega < 6 - \sqrt{20}$

$\approx 10.4721 \qquad \approx 1.5279$

None of the ω in Ω satisfies this condition.

$\omega = 0$ or 1 ← Same

Therefore, $P[Z > 20] = P(\{0,1\}) = \frac{1}{10} + \frac{1}{10} = \frac{1}{5}$.

Method 2: Because Ω is not large, it is possible to find $Z(\omega)$ for all ω.

ω	$\omega - 6$	$(\omega - 6)^2$
0	-6	36
1	-5	25
2	-4	16
3	-3	9
4	-2	4
5	-1	1
6	0	0
7	1	1
8	2	4
9	3	9

> 20