ECS 315: In-Class Exercise # 10 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

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1) Consider three events A, B, and C. Suppose

$$P\left(A^c \cap B \cap C\right) = \frac{1}{60}, \quad P\left(A \cap B^c \cap C\right) = \frac{7}{120}, \quad P\left(A \cap B \cap C^c\right) = \frac{1}{10},$$

$$P\left(A^c \cap B^c \cap C\right) = \frac{13}{120}, \quad P\left(A^c \cap B \cap C^c\right) = \frac{3}{20}, \quad P\left(A \cap B^c \cap C^c\right) = \frac{11}{40}, \text{ and}$$

$$P\left(A \cap B \cap C\right) = \frac{1}{15}$$

a) Calculate the probability that exactly one of the three events occurs.

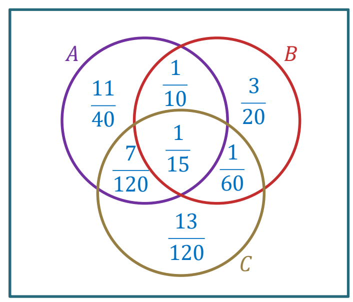
We have seen in Chapter 2 that, when we have many events $A_1, A_2, \ldots$, the event that exactly one of them occurs is the disjoint union:

$$\bigcup_k \left(A_k \cap \left(\bigcap_{i \neq k} A_i^c \right) \right).$$

Here, we have three events: A, B, and C. The probability that exactly one of the three events occurs is

$$P(A \cap B^c \cap C^c) + P(A^c \cap B \cap C^c) + P(A^c \cap B^c \cap C) = \frac{11}{40} + \frac{3}{20} + \frac{13}{120} = \frac{8}{15} \approx 0.5333.$$

b) Calculate the probability that none of the three events occurs.



We want to find $P(A^c \cap B^c \cap C^c)$ which is not given.

However, from the Venn diagram and the fact that $P(\Omega) = 1$, observe that this probability can be found by subtracting the sum of the given areas (probabilities) from the total area (1).

Therefore, $P(A^c \cap B^c \cap C^c) = 1 - \left(\frac{1}{60} + \frac{7}{120} + \frac{1}{10} + \frac{13}{120} + \frac{3}{20} + \frac{11}{40} + \frac{1}{15}\right) = 1 - \frac{31}{40} = \frac{9}{40} = 0.225.$

c) Are A, B, and C pairwise independent?

$$P(A) = \frac{11}{40} + \frac{1}{10} + \frac{7}{120} + \frac{1}{15} = \frac{1}{2} \qquad P(A \cap B) = \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$$

$$P(B) = \frac{1}{10} + \frac{1}{15} + \frac{3}{20} + \frac{1}{60} = \frac{1}{3} \qquad P(A \cap C) = \frac{7}{120} + \frac{1}{15} = \frac{1}{8}$$

$$P(C) = \frac{7}{120} + \frac{1}{15} + \frac{1}{60} + \frac{13}{120} = \frac{1}{4} \qquad P(B \cap C) = \frac{1}{60} + \frac{1}{15} = \frac{1}{12}$$

Checking pairwise independence for three events requires three conditions:

$$P(A \cap B) = P(A)\,P(B) \qquad P(A \cap C) = P(A)P(C) \qquad P(B \cap C) = P(B)P(C)$$

All three conditions are satisfied. Therefore, yes, the three events are pairwise independent.

d) Are A, B, and C independent?

Checking independence for three events requires four conditions. The first three conditions are the same as those for the pairwise independence which we have already checked in the previous part. Therefore, we only need to check the last condition:

$$P(A \cap B \cap C) \stackrel{?}{=} P(A)P(B)P(C).$$

Here, $P(A \cap B \cap C) = \frac{1}{15}$. However, $P(A)P(B)P(C) = \frac{1}{24}$. Therefore, the last condition fails. So, no, the three events are not independent.