ECS 315: In-Class Exercise # 1 - Sol

Date: 2 0 / 0 8 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three students each.
2. **Explanation is not required for this exercise.**
3. **Do not panic.**

1. (6 pt) A student played the Monty Hall game six times. Each time, he always chose to switch his choice at the last step. The results were recorded below with W for winning (getting a car) and L for losing (getting a goat).

L W L W L L.

Let A be the event that he won.

Let $R(A,n)$ denote the **relative frequency** of event A for the first n plays.

Calculate $R(A,n)$ from n = 1 to n = 6. Write your answers in the form X.XX.

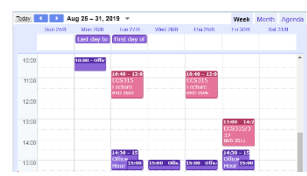
Recall that, to find the relative frequency of an event A for the first n trials, we use the formula:

$$R(A,n) = \frac{N(A,n)}{n} = \frac{\#\text{trials that } A \text{ occurs}}{n}$$

n	1	2	3	4	5	6
$R(A,n)$	$\frac{0}{1} = 0.00$	$\frac{1}{2} = 0.50$	$\frac{1}{3} \approx 0.33$	$\frac{2}{4} = 0.50$	$\frac{2}{5} = 0.40$	$\frac{2}{6} \approx 0.33$

Note how the denominator is increased as we increase the value of n.

2. (3 pt) List **all** of Dr.Prapun's office hours during next week.
Hint: Check Google Calendar on the course website.
Note that there are two types of office hours: normal and document signing. Put both types in the table below. If there are overlapping office-hour slots, "union" their time intervals first before putting the result in the table below.



Many students are uncertain or shy about utilizing a professor's office hours. Don't let that be you! Each professor should hold office hours, a designated time in which the professor is available in his or her office to speak individually with students.

Date	Time	
	From	To
August 26, 2019	10:00	10:40
August 27, 2019	14:30	15:30
August 28, 2019	15:00	15:30
August 29, 2019	15:00	15:30
August 30, 2019	14:30	15:30

3. (1 pt) (This one final task has to be worked on individually, not as a group.)
Use your own Line account to send

your student id, followed by your full name, and then your nickname inside the parentheses

into the **ECS315 Line group**.

Example: "50764555 Nadech Kugimiya (Barry)"

Remark: If you don't have a Line account, email the message as instructed above to prapun@siit.tu.ac.th.

ECS 315: In-Class Exercise # 2 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as your group** for the previous exercise.
2. [ENRE] = Explanation is not required for this exercise.
3. **Do not panic.**

Date: 2 2 / 0 8 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

[ENRE] Let

$\mathbb{N}=$ the set of all natural numbers,

$A=$ the interval $[0,2]$,

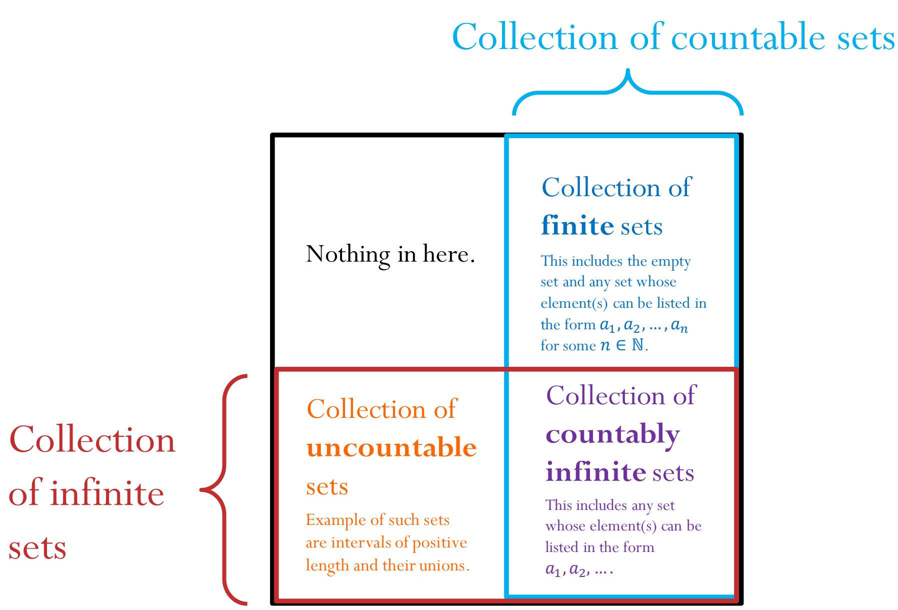
$B=$ the set of all real-valued x satisfying $\cos(x)=x^2+2$, and

$C=$ the set of all real-valued x satisfying $\cos(x)\geq 0.5$.

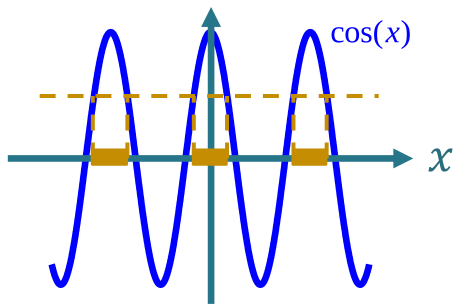
For each of the sets provided in the first column of the table below, indicate (by putting a Y(es) or an N(o) in each corresponding cell) whether it is "finite", "infinite", "countably infinite", "uncountable".

	Finite	Infinite	Countably Infinite	Uncountable
$\mathbb{N}$	N	Y	Y	N
$\{1,2,\ldots,10^{10}\}$	Y	N	N	N
A	N	Y	N	Y
B	Y	N	N	N
C	N	Y	N	Y

First, we find the "key" type of each given set. (Figure 4 from the lecture notes is copied below.)



- $\mathbb{N} = \{1,2,3,\ldots\}$ is <u>countably infinite</u>. We used it as the first example of countable infinite sets. In particular, its members can be listed in the form $a_1, a_2, a_3, \ldots$ by setting $a_k = k$.
- Although it has a lot of members, the set $\{1,2,\ldots,10^{10}\}$ is still <u>finite</u>. Its size is 10^{10} which is large but not ∞.
- Any interval with positive length is an <u>uncountable</u> set. Therefore, A is uncountable.
- Note that $\cos(x) \le 1$ but $x^2 + 2 \ge 2$. Therefore, the two functions will never intersect. Therefore, $B = \varnothing$ which is <u>finite</u>.
- For set C, one can try to make a lousy plot of $\cos(x)$ and locate the x values that give $\cos(x) \ge 0.5$. This is shown below:



Observe that these x values correspond to a union of intervals all of which have positive length. Therefore, C is <u>uncountable</u>.

	Finite	Infinite	Countably Infinite	Uncountable
$\mathbb{N}$			Y	
$\{1,2,\ldots,10^{10}\}$	Y			
A				Y
B	Y			
C				Y

Then, we can apply the following reasoning:

- Any <u>countably infinite</u> set is, by definition, infinite and hence not finite.
  Furthermore, any countably infinite set is, by definition, countable and hence not uncountable.
  So, the answers for the corresponding row are N Y <u>Y</u> N.
- Any <u>finite</u> set cannot be infinite, countably infinite, nor uncountable.
  So, the answers for the corresponding row are <u>Y</u> N N N.
- Any <u>uncountable</u> set is infinite. Any infinite set is not finite.
  Furthermore, any uncountable set is, by definition, not countable and therefore cannot be countably infinite.
  So, the answers for the corresponding row are N Y N <u>Y</u>.

ECS 315: In-Class Exercise # 3

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 27 / 08 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

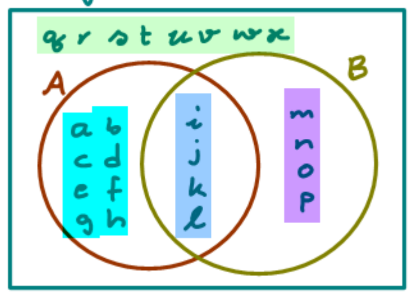
1) A random experiment has 24 **equiprobable** outcomes:

$\Omega = \{a,b,c,d,e,f,g,h,i,j,k,l,m,n,o,p,q,r,s,t,u,v,w,x\}$

Let A denote the event $\{a,b,c,d,e,f,g,h,i,j,k,l\}$, and let B denote the event $\{i,j,k,l,m,n,o,p\}$.

Determine the following:

The size of any set can be counted directly once we obtain the Venn diagram.



The solution

(a) $P(A) = \frac{|A|}{|\Omega|} = \frac{12}{24} = \frac{1}{2}$

(b) $P(A \cup B^c) = \frac{|A \cup B^c|}{|\Omega|} = \frac{|\Omega| - |(A\cup B^c)^c|}{|\Omega|} = 1 - \frac{|A^c \cap B|}{|\Omega|} = 1 - \frac{|B \setminus A|}{|\Omega|}$

$= 1 - \frac{|\{m,n,o,p\}|}{24}$

$= 1 - \frac{4}{24} = \frac{5}{6} \approx 0.8333$

Alternatively, $B^c = \{a,b,c,d,e,f,g,h,q,r,s,t,u,v,w,x\}$

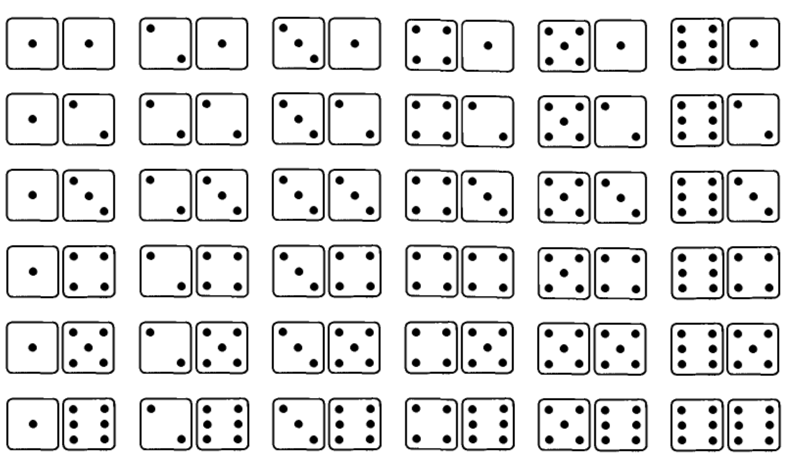
$A \cup B^c = \{a,b,c,d,e,f,g,h,i,j,k,l,q,r,s,t,u,v,w,x\}$

8 outcomes, 4 outcomes, 8 outcomes

$|A \cup B^c| = 20$

$P(A \cup B^c) = \frac{|A \cup B^c|}{|\Omega|} = \frac{20}{24} = \frac{5}{6}$

2) Roll two (fair) dice. What is the probability that the sum is greater than 9?



Let A be the event that the sum is greater than 9.

Consider all possible outcomes. The corresponding sums are

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

$|\Omega|$ $|A|$

Among the 36 outcomes, only 6 of them satisfy the condition "sum > 9".

Therefore,

$P(A) = \frac{|A|}{|\Omega|} = \frac{6}{36} = \frac{1}{6}$

ECS 315: In-Class Exercise # 4

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 29/ 08/ 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

In each of the parts below, find $P(A)$, $P(B)$, and $P(A \cap B)$.

(a) $P(A^c) = 0.5$, $P(A \cup B) = 0.6$, and $P(B^c) = 0.7$.

$P(A) = 1 - P(A^c) = 1 - 0.5 = 0.5$

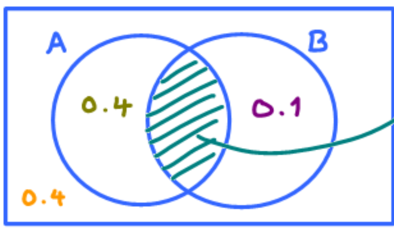
$P(B) = 1 - P(B^c) = 1 - 0.7 = 0.3$

From (5.16), $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Therefore, $P(A \cap B) = P(A) + P(B) - P(A \cup B) = 0.5 + 0.3 - 0.6 = 0.2$

$P(A) =$ 0.5, $P(B) =$ 0.3, and $P(A \cap B) =$ 0.2.

(b) $P(A^c \cap B^c) = 0.4$, $P(A \cap B^c) = 0.4$, and $P(A^c \cap B) = 0.1$.



We know that $P(\Omega) = 1$

Here, we must have

$0.4 + 0.4 + P(A \cap B) + 0.1 = 1.$

Therefore, $P(A \cap B) = 0.1$

$P(A) = P(A \cap B^c) + P(A \cap B)$
$= 0.4 + 0.1$
$= 0.5$

$P(B) = P(A \cap B) + P(A^c \cap B)$
$= 0.1 + 0.1$
$= 0.2$

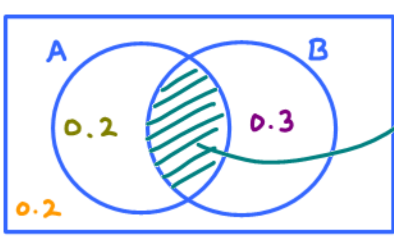
$P(A) =$ 0.5, $P(B) =$ 0.2, and $P(A \cap B) =$ 0.1.

(c) $P(A \cup B^c) = 0.7$, $P(A^c \cup B) = 0.8$, $P(A \cup B) = 0.8$.

$P(A^c \cap B^c) = 1 - P(A \cup B) = 1 - 0.8 = 0.2$

$P(A^c \cap B) = 1 - P(A \cup B^c) = 1 - 0.7 = 0.3$

$P(A \cap B^c) = 1 - P(A^c \cup B) = 1 - 0.8 = 0.2$



We know that $P(\Omega) = 1$

Here, we must have

$0.2 + 0.2 + P(A \cap B) + 0.3 = 1.$

Therefore, $P(A \cap B) = 0.3$

$P(A) = P(A \cap B^c) + P(A \cap B)$
$= 0.2 + 0.3$
$= 0.5$

$P(B) = P(A \cap B) + P(A^c \cap B)$
$= 0.3 + 0.3$
$= 0.6$

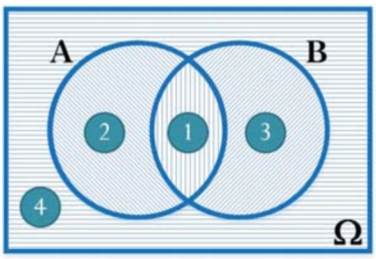
$P(A) =$ 0.5, $P(B) =$ 0.6, and $P(A \cap B) =$ 0.3.

Tips for Finding Event-Based Probability

- Don't forget that we always have an extra piece of information: $P(\Omega) = 1$.
- It is easier to work with expression involving intersection than the one with union.
  - Use de Morgan law [2.5] and complement rule [5.15]
  - For example, suppose we are given that $P(A \cup B^c) = 0.3$.
    - By the complement rule, $P\big((A \cup B^c)^c\big) = 1 - 0.3 = 0.7$.
    - By de Morgan law, $(A \cup B^c)^c = A^c \cap B$.
    - Therefore, the provided information is equivalent to $P(A^c \cap B) = 0.7$.

Tips for Finding Event-Based Probability

- Given n events, the sample space (Ω) can be partitioned into 2^n parts where each part is an intersection of the events or their complements.
- For example, when we have two events, the sample space can be partitioned into 4 parts:
  ❶ $A \cap B$,
  ❷ $A \cap B^c$,
  ❸ $A^c \cap B$, and
  ❹ $A^c \cap B^c$
  as shown in the Venn diagram.



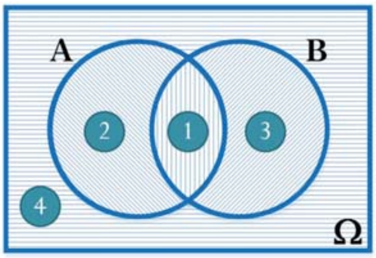
- Any event can be written as a disjoint union of these parts. Therefore, if we can find the probabilities for these parts, then we can find the probability for any event by adding the probabilities of the corresponding parts.

Tips for Finding Event-Based Probability

- If your aim is simply to find one working method to solve a problem (not trying to find the smart way to solve it), then the steps on the next slide will be helpful.
- It turns the problem into solving system of linear equations.

Steps to Find Event-Based Probability

- Let n be the number of events' names used in the question.
  - For example, if the question only talks about A and B, then $n = 2$.
- Partition the sample space (Ω) into 2^n parts where each part is an intersection of the events or their complements.
  - For example, when we have two events, the sample space can be partitioned into 4 parts:
    ❶ $A \cap B$,
    ❷ $A \cap B^c$,
    ❸ $A^c \cap B$, and
    ❹ $A^c \cap B^c$
    as shown in the Venn diagram.



- Let p_i be the probability of the i^{th} part.

Steps to Find Event-Based Probability

- Turn the given information into equation(s) of the p_i.
  - For example, if you are given that $P(A \cup B) = 0.3$, we see that $A \cup B$ cover parts ❶, ❷, and ❸. Therefore, by finite additivity, the corresponding equation is $p_1 + p_2 + p_3 = 0.3$.
  - It is easier to work with expression involving intersection than the one with union.
    - Use de Morgan law [2.5] and complement rule [5.15]
    - For example, suppose we are given that $P(A \cup B^c) = 0.3$.
      - By the complement rule, $P\left((A \cup B^c)^c\right) = 1 - 0.3 = 0.7$.
      - By de Morgan law, $(A \cup B^c)^c = A^c \cap B$.
      - Therefore, the provided information is equivalent to $P(A^c \cap B) = 0.7$.
      - The corresponding equation is $p_3 = 0.7$.
  - Don't forget that we always have an extra piece of information: $P(\Omega) = 1$.
    - With two events, this means $p_1 + p_2 + p_3 + p_4 = 1$.

Steps to Find Event-Based Probability

- Solve for the values of the p_i.
  - Note that there are n unknowns; so we will need n equations to solve for the values of the p_i.
  - If we don't have enough equations, you may be overlooking some given piece(s) of information or it is possible that you don't need to know the values of all the p_i to find the final answer(s).
- The probability of any event can be found by adding the probabilities of the corresponding parts.

Here, we follow "Steps to Find Event-Based Probability" in the slides.

ECS 315: In-Class Exercise # 4

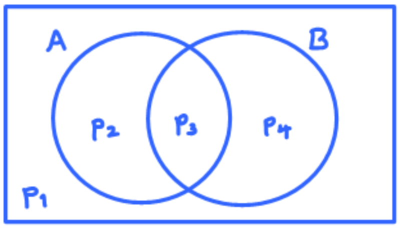
Date: 29 / 08 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

In each of the parts below, find $P(A)$, $P(B)$, and $P(A \cap B)$.

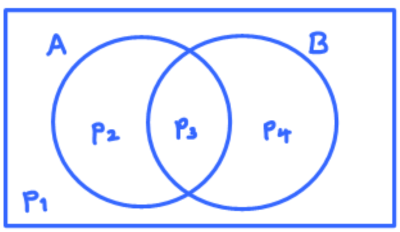
(a) $P(A^c) = 0.5$, $P(A \cup B) = 0.6$, and $P(B^c) = 0.7$.



$p_1 + p_4 = 0.5$
$p_1 + p_2 = 0.7$
$p_2 + p_3 + p_4 = 0.6$
$p_1 + p_2 + p_3 + p_4 = 1$
$\Rightarrow$ $p_1 = 0.4$, $p_2 = 0.3$, $p_3 = 0.2$, $p_4 = 0.1$

$P(A) =$ 0.5 ($p_2 + p_3$), $P(B) =$ 0.3 ($p_3 + p_4$), and $P(A \cap B) =$ 0.2 (p_3).

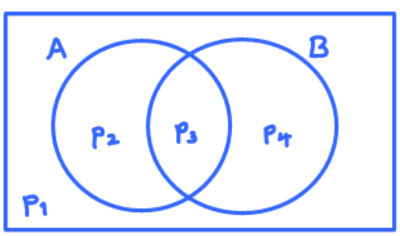
(b) $P(A^c \cap B^c) = 0.4$, $P(A \cap B^c) = 0.4$, and $P(A^c \cap B) = 0.1$.



$p_1 = 0.4$
$p_2 = 0.4$
$p_4 = 0.1$
$p_1 + p_2 + p_3 + p_4 = 1 \Rightarrow p_3 = 1 - 0.4 - 0.4 - 0.1 = 0.1$

$P(A) =$ 0.5 ($p_2 + p_3$), $P(B) =$ 0.2 ($p_3 + p_4$), and $P(A \cap B) =$ 0.1 (p_3).

(c) $P(A \cup B^c) = 0.7$, $P(A^c \cup B) = 0.8$, $P(A \cup B) = 0.8$.



$p_2 + p_3 + p_4 = 0.8$
$p_1 + p_2 + p_3 = 0.7$
$p_1 + p_3 + p_4 = 0.8$
$p_1 + p_2 + p_3 + p_4 = 1$
$p_1 = 0.2$, $p_2 = 0.2$, $p_3 = 0.3$, $p_4 = 0.3$

$P(A) =$ 0.5 ($p_2 + p_3$), $P(B) =$ 0.6 ($p_3 + p_4$), and $P(A \cap B) =$ 0.3 (p_3).

ECS 315: In-Class Exercise # 5

Date: 30 / 08 / 2019	
Name	ID (last 3 digits)
Prapun	5 5 5

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. Each answer should be reduced into just an integer.
   Exception: When the answer is more than 10 9 , you may leave the answer in some form of simplified expression.
4. **Do not panic.**

1. Calculate the following quantities:
   a. $5! = 5\times4\times3\times2\times1 = 20\times6 = 120$
   b. $\binom{8}{5} = \frac{8!}{5!3!} = \frac{8\times7\times6}{3\times2\times1} = 56$
   c. $(5)_3 = 5\times4\times3 = 60$
   d. $\binom{8}{1,2,5} = \frac{8!}{5!2!1!} = \frac{8\times7\times6}{2\times1} = 168$

2. Suppose we sample 5 objects from a collection of 8 distinct objects. Calculate the number of different possibilities when
   a. the sampling is ordered and performed with replacement

   $8^5 = 32,768$

   b. the sampling is ordered and performed without replacement

   $8\times7\times6\times5\times4 = 6,720$

   c. the sampling is unordered and performed without replacement

   $\binom{8}{5} = 56$

3. Calculate the number of different results when we permute
   a. ABC

   $3! = 3\times2\times1 = 6$

   b. AABBCC

   $\frac{6!}{2!2!2!} = \frac{6\times5\times4\times3}{2\times2} = 90$

Don't forget to simplify your answers.

ECS 315: In-Class Exercise # 6 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. **Unless specified otherwise, write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 0 3 / 0 9 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

1. Consider a random experiment whose sample space is $\{a, b, c, d, e\}$ with probabilities 0.1, 0.2, 0.2, 0.2, 0.3, respectively.

Here, it is given that $P(\{a\}) = 0.1$, $P(\{b\}) = 0.2$, $P(\{c\}) = 0.2$, $P(\{d\}) = 0.2$, $P(\{e\}) = 0.3$.

Let $A = \{a, b, c\}$, $B = \{b, c, d\}$, and $C = \{c, d, e\}$.

Find the following probabilities.

$P(A) = P(\{a,b,c\}) = P(\{a\}) + P(\{b\}) + P(\{c\}) = 0.1 + 0.2 + 0.2 = 0.5$	$P(B) = P(\{b,c,d\}) = P(\{b\}) + P(\{c\}) + P(\{d\}) = 0.2 + 0.2 + 0.2 = 0.6$
$P(A \cap B) = P(\{b,c\}) = P(\{b\}) + P(\{c\}) = 0.2 + 0.2 = 0.4$	$P((A \cup B) \cap C) = P(\{a,b,c,d\} \cap \{c,d,e\}) = P(\{c,d\}) = P(\{c\}) + P(\{d\}) = 0.2 + 0.2 = 0.4$
$P(A\|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.4}{0.6} = \frac{2}{3}$	$P(A^c\|B) =$ $A^c \cap B = \{d,e\} \cap \{b,c,d\} = \{d\}$ $= \frac{P(A^c \cap B)}{P(B)} = \frac{P(\{d\})}{P(B)} = \frac{0.2}{0.6} = \frac{1}{3}$
$P(A^c\|B^c) =$ $A^c \cap B^c = \{d,e\} \cap \{a,e\} = \{e\}$ $= \frac{P(A^c \cap B^c)}{P(B^c)} = \frac{P(\{e\})}{1 - P(B)} = \frac{0.3}{0.4} = \frac{3}{4}$	$P((A \cap C)\|B) = \frac{P(A \cap B \cap C)}{P(B)} = \frac{P(\{c\})}{P(B)} = \frac{0.2}{0.6} = \frac{1}{3}$

ECS 315: In-Class Exercise # 7 - Sol

Date: 0 5 / 0 9 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. Explanation is **not required** for this exercise. [ENRE]
3. **Do not panic.**

1) Consider the following sequences of 1s and 0s which summarize the data obtained from 15 testees in a disease testing experiment.

```
 D:  0 1 1 0 0 0 0 1 1 1 0 1 0 1 1
       1 2         3 4 5   6   7 8
TP:  0 0 0 0 1 0 1 1 0 0 1 0 0 1 1
             1   2 3     4     5 6
```

The results in the *i*-th column are for the *i*-th testee. The D row indicates whether each of the testees actually has the disease under investigation. The TP row indicates whether each of the testees is tested positive for the disease. Numbers "1" and "0" correspond to "True" and "False", respectively.

Suppose we randomly pick a testee from this pool of 15 testees. Let D be the event that the selected person actually has the disease. Let T_P be the event that the selected person is tested positive for the disease. Find the following probabilities. No explanation is needed here.

There are 15 testees; so the sample space is finite. We "randomly" pick one testee; so it makes sense to assume that each testee has equal chance of being selected. Therefore, classical probability can be applied here.

$P(D)=\frac{8}{15}$ Among the 15 testees, 8 have the disease.	$P(T_P)=\frac{6}{15}=\frac{2}{5}$ Among the 15 testees, 6 test positive.
$P(T_P\cap D)=\frac{3}{15}=\frac{1}{5}$ Among the 15 testees, 3 have the disease and test positive.	$P(T_P\cap D^c)=\frac{3}{15}=\frac{1}{5}$ Among the 15 testees, 3 test positive but do not have the disease.

In each part below, additional information about the selected testee is available; this additional information is given in the condition part. With such information, find the corresponding conditional probability.

$P(T_P\mid D)=\frac{3}{8}$ Among the 8 testees who have the disease, 3 have the disease.	$P(T_P\mid D^c)=\frac{3}{7}$ Among the $15-8=7$ testees who don't have the disease, 3 test positive.

Alternatively,

$$P(T_P|D)=\frac{P(T_P\cap D)}{P(D)}=\frac{\frac{1}{5}}{\frac{8}{15}}=\frac{3}{8}.$$

$$P(T_P|D^c)=\frac{P(T_P\cap D^c)}{P(D^c)}=\frac{\frac{1}{5}}{1-P(D)}=\frac{\frac{1}{5}}{1-\frac{8}{15}}=\frac{3}{7}.$$

ECS 315: In-Class Exercise # 8 - Sol

Date: 1 2 / 0 9 / 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

$P(\text{HIV}) = \frac{1}{25}$

Suppose that for Westeros, 1 in 25 people carries the human immunodeficiency virus (HIV). A test for the presence of HIV yields either a positive (+) or negative (-) response. Suppose the test gives the correct answer 75% of the time. (The test is 75% accurate.)

This part of the problem gives two conditional probabilities:

$P(+|\text{HIV}) = P(-|\text{HIV}^c) = 0.75 = \frac{3}{4}$

(a) What is P(−|HIV), the conditional probability that a randomly-chosen person tests negative given that the person does have the HIV virus?

Recall that $P(A^c|B) = 1 - P(A|B)$.

Therefore, $P(-|\text{HIV}) = 1 - P(+|\text{HIV}) = 1 - \frac{3}{4} = \frac{1}{4}$.

(b) Find the probability that a randomly-chosen person tests positive.

By the total probability theorem,

$$P(+) = P(+|\text{HIV})P(\text{HIV}) + P(+|\text{HIV}^c)P(\text{HIV}^c)$$
$$= \frac{3}{4} \times \frac{1}{25} + \frac{1}{4} \times \frac{24}{25} = \frac{27}{100} = 0.27.$$

$$P(+|\text{HIV}^c) = 1 - P(-|\text{HIV}^c) = 1 - \frac{3}{4} = \frac{1}{4}.$$

(c) Find the conditional probability that a randomly-chosen person has the HIV virus given that the person tests positive.

By "Form 1" of the Bayes' theorem,

$$P(\text{HIV}|+) = \frac{P(+|\text{HIV})P(\text{HIV})}{P(+)} = \frac{\frac{3}{4} \times \frac{1}{25}}{\frac{27}{100}} = \frac{3}{27} = \frac{1}{9} \approx 0.11.$$

ECS 315: In-Class Exercise # 9 - Sol

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 1 7 / 0 9 / 2019			
Name	ID (last 3 digits)		

(1) Suppose $P(A)=\frac{1}{2}$ and $P(B)=\frac{1}{6}$.

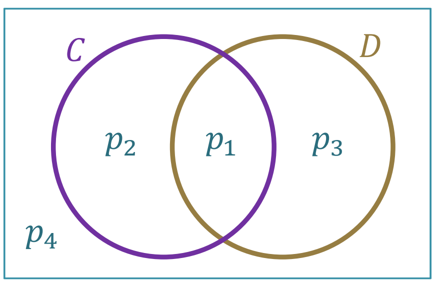
Find $P(A\cap B)$ to make events A and B independent.

By definition, events A and B are independent if and only if $P(A\cap B)=P(A)P(B)$.
Therefore, $P(A\cap B)=\frac{1}{2}\times\frac{1}{6}=\frac{1}{12}$.

(2) Suppose $P(C)=\frac{1}{4}$ and $P(C\cup D)=\frac{5}{8}$.

Find $P(C\cap D)$ to make events C and D independent.

We will use a systematic approach. Consider the Venn diagram below.



As usual, for two events we partition the sample space (Ω) into 4 parts.

Let p_i be the probability of the i^{th} part.

Observe that $P(C\cap D)$ is p_1.

From the provided information, we know that

$$p_1+p_2=P(C)=\frac{1}{4}. \quad (1)$$

$$p_1+p_2+p_3=P(C\cup D)=\frac{5}{8}. \quad (2)$$

As usual, we also know that

$$p_1+p_2+p_3+p_4=1. \quad (3)$$

The requirement that events C and D must be independent means

$$P(C\cap D)=P(C)P(D)$$

This is equivalent to

$$p_1=(p_1+p_2)(p_2+p_3). \quad (4)$$

Note that we now have four equations to solve for four unknowns. This should be possible to do.
Here, it requires only a few more steps to solve for p_1.

$$p_1=\frac{1}{4}\left(\frac{5}{8}-p_1\right).$$

$$p_1=\frac{1}{8}.$$

There are usually other “easier” solutions. However, they are not as systematic as the solution above.

For example, one can start with

$$P(C \cup D) = P(C) + P(D) - P(C \cap D).$$

Forcing the events C and D to be independent means we must have $P(C \cap D) = P(C)P(D)$.

So,

$$P(C \cup D) = P(C) + P(D) - P(C)P(D).$$

Plugging in the provided values, we have

$$\frac{5}{8} = \frac{1}{4} + P(D) - \frac{1}{4}P(D).$$

This gives

$$P(D) = \frac{1}{2}.$$

Therefore,

$$P(C \cap D) = P(C)P(D) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}.$$

ECS 315: In-Class Exercise # 10 - Sol

Date: 19/ 09/ 2019			
Name	ID (last 3 digits)		
Prapun	5	5	5

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1) Consider three events A, B, and C. Suppose

$$P\left(A^c \cap B \cap C\right) = \frac{1}{60},\quad P\left(A \cap B^c \cap C\right) = \frac{7}{120},\quad P\left(A \cap B \cap C^c\right) = \frac{1}{10},$$

$$P\left(A^c \cap B^c \cap C\right) = \frac{13}{120},\quad P\left(A^c \cap B \cap C^c\right) = \frac{3}{20},\quad P\left(A \cap B^c \cap C^c\right) = \frac{11}{40}, \text{ and}$$

$$P\left(A \cap B \cap C\right) = \frac{1}{15}$$

a) Calculate the probability that exactly one of the three events occurs.

We have seen in Chapter 2 that, when we have many events $A_1, A_2, \ldots$, the event that exactly one of them occurs is the disjoint union:

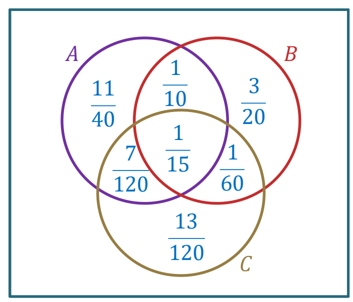
$$\bigcup_k \left(A_k \cap \left(\bigcap_{i \neq k} A_i^c \right) \right).$$

Here, we have three events: A, B, and C. The probability that exactly one of the three events occurs is

$$P(A \cap B^c \cap C^c) + P(A^c \cap B \cap C^c) + P(A^c \cap B^c \cap C) = \frac{11}{40} + \frac{3}{20} + \frac{13}{120} = \frac{8}{15} \approx 0.5333.$$

b) Calculate the probability that none of the three events occurs.

We want to find $P(A^c \cap B^c \cap C^c)$ which is not given.
However, from the Venn diagram and the fact that $P(\Omega) = 1$, observe that this probability can be found by subtracting the sum of the given areas (probabilities) from the total area (1).



Therefore, $P(A^c \cap B^c \cap C^c) = 1 - \left(\frac{1}{60} + \frac{7}{120} + \frac{1}{10} + \frac{13}{120} + \frac{3}{20} + \frac{11}{40} + \frac{1}{15}\right) = 1 - \frac{31}{40} = \frac{9}{40} = 0.225.$

c) Are A, B, and C pairwise independent?

$$P(A) = \frac{11}{40} + \frac{1}{10} + \frac{7}{120} + \frac{1}{15} = \frac{1}{2} \qquad P(A \cap B) = \frac{1}{10} + \frac{1}{15} = \frac{1}{6}$$

$$P(B) = \frac{1}{10} + \frac{1}{15} + \frac{3}{20} + \frac{1}{60} = \frac{1}{3} \qquad P(A \cap C) = \frac{7}{120} + \frac{1}{15} = \frac{1}{8}$$

$$P(C) = \frac{7}{120} + \frac{1}{15} + \frac{1}{60} + \frac{13}{120} = \frac{1}{4} \qquad P(B \cap C) = \frac{1}{60} + \frac{1}{15} = \frac{1}{12}$$

Checking pairwise independence for three events requires three conditions:

$$P(A \cap B) = P(A)\,P(B) \qquad P(A \cap C) = P(A)P(C) \qquad P(B \cap C) = P(B)P(C)$$

All three conditions are satisfied. Therefore, yes, the three events are pairwise independent.

d) Are A, B, and C independent?

Checking independence for three events requires four conditions. The first three conditions are the same as those for the pairwise independence which we have already checked in the previous part. Therefore, we only need to check the last condition:

$$P(A \cap B \cap C) \stackrel{?}{=} P(A)P(B)P(C).$$

Here, $P(A \cap B \cap C) = \frac{1}{15}$. However, $P(A)P(B)P(C) = \frac{1}{24}$. Therefore, the last condition fails. So, no, the three events are not independent.