ECS 315: In-Class Exercise # 11

Instructions

1. Separate into groups of no more than three students each.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: _ _ / _ _ / 2019			
Name	ID (last 3 digits)		

1. Consider a random experiment in which you roll a six-sided fair dice (whose faces are numbered 1-6). We define the following random variables from the outcomes of this experiment:

$$X(\omega) = \omega \quad \text{and} \quad Y(\omega) = 2 + \left((\omega-1)(\omega-3)(\omega-5)(\omega-7)\right).$$

   a. Find $P[X = 2]$.

   b. Find $P[Y = 2]$.

2. Consider a random experiment in which you roll a 10-sided fair dice (whose faces are numbered 0–9). Define a random variable Z from the outcomes of this experiment by

$$Z(\omega) = (\omega - 6)^2.$$

   a. Find $P[Z = 4]$.

   b. Find $P[Z > 20]$.

ECS 315: In-Class Exercise # 12

Date: _ _ / _ _ / 2019			
Name	ID (last 3 digits)		

Instructions

1. Separate into groups of no more than three students each.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Consider a random variable whose pmf is given by $p_X(x) = \begin{cases} \frac{1}{4}, & x = 1, 9, \\ c, & x = 5, \\ 0, & \text{otherwise.} \end{cases}$

a) Find the constant c.

b) Plot $p_X(x)$. (Recall that we use stem plot for pmf.)

c) Find $P[X \le 7]$.

d) Find $P[X > 4]$.

e) Find $P[X \le 5]$.

f) Find $P[X \le 4.99]$.

g) Find $P[X \le 5.01]$.

ECS 315: In-Class Exercise # 13

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. [ENRE] Explanation is not required for this exercise.
3. **Do not panic.**

Date: _ _ / _ _ / 2019			
Name	ID (last 3 digits)		

1. Consider a random variable X whose pmf is given by

$$p_X(x) = \begin{cases} 0.2, & x = -1, \\ c, & x = 1, 3, \\ 0, & \text{otherwise.} \end{cases}$$

   a. Find the constant c.

   b. Plot the cdf of this random variable.

2. Consider a random variable X whose cdf is given by

$$F_X(x) = \begin{cases} 0, & x < 0, \\ 0.2, & 0 \le x < 3, \\ 1, & x \ge 3. \end{cases}$$

   a. Find $P[X \le 1]$.

   b. Find $P[X > 1]$.

   c. Plot the pmf of X.

ECS 315: In-Class Exercise # 14

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. [ENRE] Explanation is not required for this exercise.
3. **Do not panic.**

Date: _ _ / _ _ / 2019			
Name	ID (last 3 digits)		

Consider the random variable specified in each part below.

i) Write down its (minimal) support.

ii) Find $P[X=0]$. Your answer should be of the form 0.XXXX.

iii) Find $P[X=2]$. Your answer should be of the form 0.XXXX.

	(minimal) support	$P[X=0]$	$P[X=2]$
$X \sim \mathcal{U}(\{-2,0,2\})$			
$X \sim \text{Bernoulli}\left(\frac{1}{4}\right)$			
$X \sim \mathcal{B}(4,0.6)$			

ECS 315: In-Class Exercise # 15

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: _ _ / _ _ / 2019			
Name	ID (last 3 digits)		

1. Suppose $X \sim \mathcal{G}_0\left(\frac{1}{3}\right)$. Find $P\left[X \le \frac{1}{3}\right]$.

2. Suppose $X \sim \mathcal{G}_1\left(\frac{1}{3}\right)$. Find $P\left[X \le \frac{1}{3}\right]$.

3. [ENRPa] Consider (a sequence of independent) Bernoulli trials whose success probability for each trial is $1/4$. For each of the random variables defined below, indicate the **name and the parameter(s)** of the family it belongs to.

Random Variable	Family
$K =$ the number of failures until the first success occurs.	
$N =$ the number of successes among the first 4 trials.	
$F =$ the number of successes until the first failure occurs.	

ECS 315: In-Class Exercise # 16

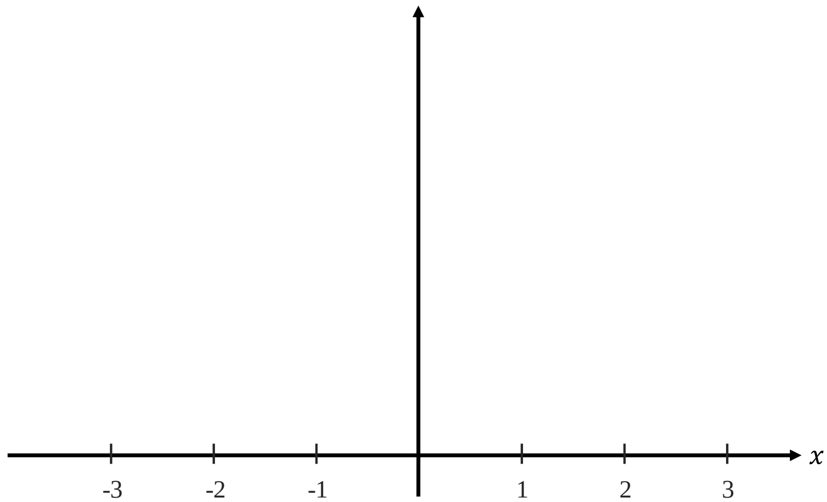
Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: _ _ / _ _ / 2019			
Name	ID (last 3 digits)		

1. Arrivals of customers at a restaurant are modeled by a Poisson process with a rate of $\lambda = \frac{1}{3}$ customers per minute. Let X be the number of customers arriving between 5:00 PM and 5:06 PM.

   a. Plot its cdf $F_X(x)$ on the interval $[-3,3]$.



   b. What is the probability that $X < 1$?

2. Let N be the number of successes in 10^{20} Bernoulli trials. Assume that the probability of success for each trial is 10^{-21}. Use Poisson approximation to calculate $P[N=0]$.

ECS 315: In-Class Exercise # 17

Date: _ _ / _ _ / 2019			
Name	ID (last 3 digits)		

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Find the expected value of the random variable X defined in each part below:

   a. $p_X(x) = \begin{cases} cx, & x \in \{1,2\}, \\ 0, & \text{otherwise.} \end{cases}$

   b. $p_X(x) = \begin{cases} 0.3, & x = -1, 1, \\ c, & x = 3, \\ 0, & \text{otherwise.} \end{cases}$

   c. $F_X(x) = \begin{cases} 0, & x < -1, \\ 0.4, & -1 \le x < 1, \\ 1, & x \ge 1. \end{cases}$

   d. $p_X(x) = \begin{cases} cx, & x \in \{1,2,3,\ldots,10\}, \\ 0, & \text{otherwise.} \end{cases}$

ECS 315: In-Class Exercise # 18

Date: _ _ / _ _ / 2019			
Name	ID (last 3 digits)		

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Find $\mathbb{E}\left[X^2\right]$ and $\operatorname{Var}\left[X\right]$ for the random variable X defined in each part below:

$p_X(x)$	$\mathbb{E}\left[X\right]$	$\mathbb{E}\left[X^2\right]$	$\operatorname{Var}\left[X\right]$
$\begin{cases} \frac{1}{3}x, & x \in \{1,2\}, \\ 0, & \text{otherwise.} \end{cases}$	$\frac{5}{3}$		
$\begin{cases} 0.3, & x = -1, 1, \\ 0.4, & x = 3, \\ 0, & \text{otherwise.} \end{cases}$	1.2		
$\begin{cases} \frac{1}{55}x, & x \in \{1,2,3,\ldots,10\}, \\ 0, & \text{otherwise.} \end{cases}$	7		

ECS 315: In-Class Exercise # 19

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 0 5 / 1 1 / 2019			
Name	ID (last 3 digits)		

In this question, we consider two distributions for a random variable X.
In part (a), which corresponds to the second column in the table below, X is a ***discrete*** random variable with its pmf specified in the first row.
In part (b), which corresponds to the third column, X is a ***continuous*** random variable with its pdf specified in the first row.

	$p_X(x) = \begin{cases} cx^2, & x \in \{-1, 2\}, \\ 0, & \text{otherwise.} \end{cases}$	$f_X(x) = \begin{cases} cx^2, & x \in (-1, 2], \\ 0, & \text{otherwise.} \end{cases}$
Find c		
Find $P[X = 2]$		
Find $P[0 < X \le 2]$		

ECS 315: In-Class Exercise # 20

Date: 0 7 / 1 1 / 2019			
Name	ID (last 3 digits)		

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. [ENRE] Explanation is not required for this exercise..
3. **Do not panic.**

In this question, we consider two distributions for a random variable X.
In part (a), which corresponds to the second column in the table below, X is a ***discrete*** random variable with its pmf specified in the first row.
In part (b), which corresponds to the third column, X is a ***continuous*** random variable with its pdf specified in the first row.

	$p_X(x) = \begin{cases} \frac{1}{5}x^2, & x \in \{-1, 2\}, \\ 0, & \text{otherwise.} \end{cases}$	$f_X(x) = \begin{cases} \frac{1}{3}x^2, & x \in (-1, 2], \\ 0, & \text{otherwise.} \end{cases}$
Find the cdf $F_X(x)$		
Plot the cdf $F_X(x)$		

ECS 315: In-Class Exercise # 21

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 1 2 / 1 1 / 2019			
Name	ID (last 3 digits)		

In this question, we consider two distributions for a random variable X.
In part (a), which corresponds to the second column in the table below, X is a ***discrete*** random variable with its pmf specified in the first row.
In part (b), which corresponds to the third column, X is a ***continuous*** random variable with its pdf specified in the first row.

	$p_X(x) = \begin{cases} \frac{1}{5}x^2, & x \in \{-1, 2\}, \\ 0, & \text{otherwise.} \end{cases}$	$f_X(x) = \begin{cases} \frac{1}{3}x^2, & x \in (-1, 2], \\ 0, & \text{otherwise.} \end{cases}$
Find $\mathbb{E}X$		
Find $\mathbb{E}\left[X^2\right]$		
Find $\text{Var}[X]$		
Find σ_X		

ECS 315: In-Class Exercise # 21

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: 1 4 / 1 1 / 2019			
Name	ID (last 3 digits)		

Calculate $P[0.5 < X \le 1.5]$ for each of the following random variables.

Your answer should be of the form 0.XXXX.

a) $X \sim \mathcal{U}(1,4)$

b) $X \sim \mathcal{E}(1)$

c) $X \sim \mathcal{N}(0,1)$

d) $X \sim \mathcal{N}(1,3)$

ECS 315: In-Class Exercise # 23

Date: 1 9 / 1 1 / 2019			
Name	ID (last 3 digits)		

Instructions

1. Separate into groups of no more than three students each. **The group cannot be the same as any of your former groups after the midterm.**
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Random variables X and Y have the following joint pmf

$$p_{X,Y}(x,y) = \begin{cases} c(x+y)^2, & x \in \{-1,2\} \text{ and } y \in \{0,-3\}, \\ 0, & \text{otherwise.} \end{cases}$$

a) Find c

b) Find the joint pmf matrix $\mathbf{P}_{X,Y}$

c) Find $P[X > Y]$.

d) Find the pmf $p_X(x)$ and the pmf $p_Y(y)$.

ECS 315: In-Class Exercise # 24

Instructions

1. Separate into groups of no more than three students each.
2. [ENRE] Explanation is not required for this exercise.
3. **Do not panic.**

Date: 2 1 / 1 1 / 2019			
Name	ID (last 3 digits)		

1) Suppose the pmf of a random variable X is given by

$$p_X(x) = \begin{cases} 0.1, & x = 2, \\ c, & x = 4, \\ 0, & \text{otherwise.} \end{cases}$$

Let Y be another random variable. Assume that X and Y are i.i.d.

Find

a) c = ________

b) Their joint pmf matrix $\mathbf{P}_{X,Y}$.

2) Random variables X and Y are independent. Their joint pmf matrix is

$$\mathbf{P}_{X,Y} = \begin{array}{c} x \backslash y \\ -1 \\ 3 \end{array} \!\!\! \begin{array}{c} \begin{array}{ccc} \;2\; & \;\;5\;\; & \;8\; \end{array} \\ \begin{bmatrix} 0.08 & 0.12 & a \\ 0.12 & b & c \end{bmatrix} \end{array}.$$

Find the values of the unknown constants:

$$a = \_\_\_\_, \quad b = \_\_\_\_, \quad c = \_\_\_\_ .$$

ECS 315: In-Class Exercise # 25

Date: 2 5 / 1 1 / 2019			
Name	ID (last 3 digits)		

Instructions

1. Separate into groups of no more than three students each.
2. Write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Random variables X and Y have the following joint pmf matrix

$$\mathbf{P}_{X,Y} = \begin{array}{c c} x \backslash y & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 1 \\ 0 \end{array} & \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & c \end{bmatrix} \end{array}$$

Are X and Y uncorrelated?

2. Random variables X and Y have the following joint pmf matrix

$$\mathbf{P}_{X,Y} = \begin{array}{c c} x \backslash y & \begin{array}{cc} 0 & 1 \end{array} \\ \begin{array}{c} 1 \\ 0 \end{array} & \begin{bmatrix} 0.1 & a \\ 0.2 & b \end{bmatrix} \end{array}$$

Suppose X and Y are uncorrelated. Find the values of the unknown constants:

$a =$ __________, $b =$ __________.