Name _____ ID3 _ _ _

ECS 315: Probability and Random Processes **2018/1**

HW 2 — Due: September 4, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 4 pages.

(b) (1 pt) Work and write your answers **directly on this sheet** (not on another blank sheet of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID in the spaces provided on the upper-right corner of this page.

(d) (8 pt) Try to solve all non-optional problems.

(e) Late submission will be heavily penalized.

Problem 1. [Montgomery and Runger, 2010, Q2-54] Each of the possible five outcomes of a random experiment is equally likely. The sample space is $\{a, b, c, d, e\}$. Let A denote the event $\{a, b\}$, and let B denote the event $\{c, d, e\}$. Determine the following:

(a) $P(A)$

(b) $P(B)$

(c) $P(A^c)$

(d) $P(A \cup B)$

(e) $P(A \cap B)$

Problem 2. (**Classical Probability and Combinatorics**) Shuffle a deck of cards and cut it into three piles. What is the probability that (at least) a court card will turn up on top of one of the piles.

Hint: There are 12 court cards (four jacks, four queens and four kings) in the deck.

Problem 3. (**Classical Probability**) There are three buttons which are painted red on one side and white on the other. If we tosses the buttons into the air, calculate the probability that all three come up the same color.

Remarks: A *wrong* way of thinking about this problem is to say that there are four ways they can fall. All red showing, all white showing, two reds and a white or two whites and a red. Hence, it seems that out of four possibilities, there are two favorable cases and hence the probability is 1/2.

Problem 4. (**Classical Probability and Combinatorics**) A Web ad can be designed from four different colors, three font types, five font sizes, three images, and five text phrases.

(a) How many different designs are possible? [Montgomery and Runger, 2010, Q2-51]

(b) A specific design is randomly generated by the Web server when you visit the site. If you visit the site five times, what is the probability that you will not see the same design? [Montgomery and Runger, 2010, Q2-71]

Problem 5. (**Classical Probability and Combinatorics**) A bin of 50 parts contains five that are defective. A sample of two parts is selected at random, without replacement. Determine the probability that both parts in the sample are defective. [Montgomery and Runger, 2010, Q2-49]

Problem 6. (**Combinatorics**) Consider the design of a communication system in the United States.

(a) How many three-digit phone prefixes that are used to represent a particular geographic area (such as an area code) can be created from the digits 0 through 9?

(b) How many three-digit phone prefixes are possible in which no digit appears more than once in each prefix?

(c) As in part (a), how many three-digit phone prefixes are possible that do not start with 0 or 1, but contain 0 or 1 as the middle digit?

[Montgomery and Runger, 2010, Q2-45]

Problem 7. (**Classical Probability and Combinatorics**) We all know that the chance of a head (H) or tail (T) coming down after a fair coin is tossed are fifty-fifty. If a fair coin is tossed ten times, then intuition says that five heads are likely to turn up.

Calculate the probability of getting exactly five heads (and hence exactly five tails).

Extra Question

Here is an optional question for those who want more practice.

Problem 8. ***An Even Split at Coin Tossing***: Let p_n be the probability of getting exactly n heads (and hence exactly n tails) when a fair coin is tossed $2n$ times.

(a) Find p_n.

(b) Sometimes, to work theoretically with large factorials, we use Stirling's Formula:

$$n! \approx \sqrt{2\pi n} n^n e^{-n} = \left(\sqrt{2\pi e}\right) e^{\left(n+\frac{1}{2}\right)\ln\left(\frac{n}{e}\right)}. \tag{2.1}$$

Approximate p_n using Stirling's Formula.

(c) Find $\lim_{n\to\infty} p_n$.