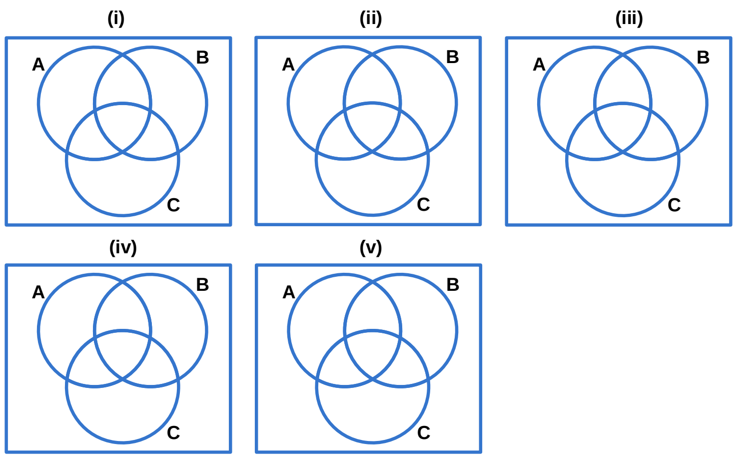
ECS 315: Probability and Random Processes **2018/1**

HW Solution 1 — Due: August 28, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Problem 1. (**Set Theory**) For this problem, only answers are needed; you don't have to describe your solution.

(a) In the Venn diagrams below,



shade the region that corresponds to the following events:

(i) A^c

(ii) $A \cap B$

(iii) $(A \cap B) \cup C$

(iv) $(B \cup C)^c$

(v) $(A \cap B)^c \cup C$

[Montgomery and Runger, 2010, Q2-19]

(b) Let $\Omega = \{0, 1, 2, 3, 4, 5, 6, 7\}$, and put $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, and $C = \{5, 6\}$. Find

(i) $A \cup B$

(ii) $A \cap B$

(iii) $A \cap C$

(iv) A^c

(v) $B \setminus A$

Solution:

(a) See Figure 1.1

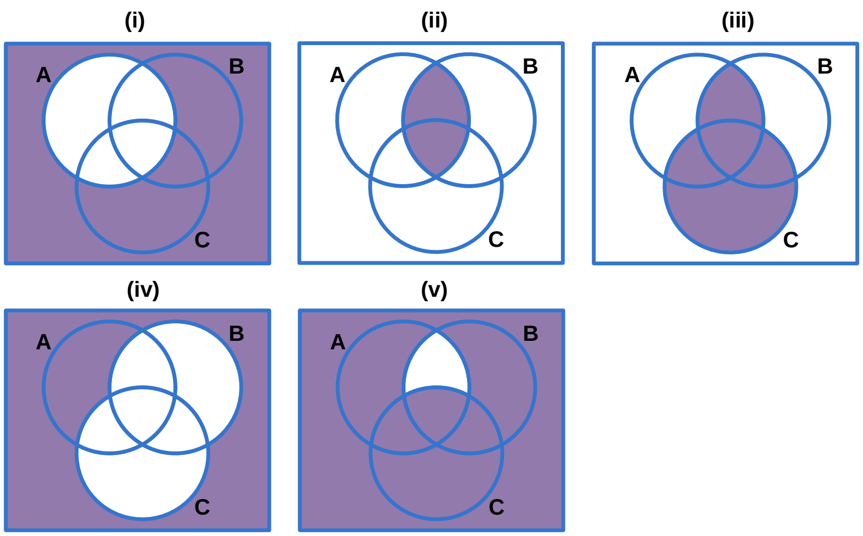


Figure 1.1: Venn diagrams for events in Problem 1

(b) $A \cup B = \{1, 2, 3, 4, 5, 6\}$, $A \cap B = \{3, 4\}$, $A \cap C = \emptyset$, $B \setminus A = \{5, 6\} = C$.

Problem 2. For this problem, only answers are needed; you don't have to provide explanation.

For each of the sets provided in the first column of the table below, indicate (by putting a Y(es) or an N(o) in the appropriate cells of the table) whether it is "finite", "infinite", "countable", "countably infinite", "uncountable".

Sets	Finite	Infinite	Countable	Countably Infinite	Uncountable
$\{1\}$					
$\{1, 2\}$					
$[1, 2]$					
$[1, 2] \cup [-1, 0]$					
$\{1, 2, 3, 4\}$					
the power set of $\{1, 2, 3, 4\}$					
the set of all real numbers					
the set of all real-valued x satisfying $\cos x = 0$					
the set of all integers					
$(-\infty, 0]$					
$(-\infty, 0] \cap [0, +\infty)$					

Solution: First, note that the intersection in the last row can be simplified into a singleton $\{0\}$. Being an intersection of intervals may make it look like an uncountable sets. However, only one number survives the intersection.

The sets $\{1\}$, $\{1, 2\}$, $\{1, 2, 3, 4\}$, $2^{\{1,2,3,4\}}$, and $\{0\}$ are all finite set because their size (cardinality) are finite. Because they are finite, they are not infinite. Any finite set is countable. So, they are countable. They are not infinite; so they can't be countably infinite nor uncountable. Their corresponding rows should be Y N Y N N.

The x that satisfies $\cos x = 0$ is any x of the form $\frac{\pi}{2} + k\pi$ where k is any integer. So, the collection of these x has the same size (cardinality) as the set of all integers. They are countably infinite which means they are infinite and countable. Because they are countable, they are not uncountable. Because they are infinite, they are not finite. Their corresponding rows should be N Y Y Y N.

The sets $[1, 2]$, $[1, 2] \cup [-1, 0]$, $\mathbb{R}$, and $(-\infty, 0]$ are all intervals and hence uncountable. Uncountable sets are not countable and hence can't be countably infinite. They are infinite and hence not finite. Their corresponding rows should be N Y N N Y.

Sets	Finite	Infinite	Countable	Countably Infinite	Uncountable
$\{1\}$	Y	N	Y	N	N
$\{1,2\}$	Y	N	Y	N	N
$[1,2]$	N	Y	N	N	Y
$[1,2] \cup [-1,0]$	N	Y	N	N	Y
$\{1,2,3,4\}$	Y	N	Y	N	N
the power set of $\{1,2,3,4\}$	Y	N	Y	N	N
the set of all real numbers	N	Y	N	N	Y
the set of all real-valued x satisfying $\cos x = 0$	N	Y	Y	Y	N
the set of all integers	N	Y	Y	Y	N
$(-\infty, 0]$	N	Y	N	N	Y
$(-\infty, 0] \cap [0, +\infty)$	Y	N	Y	N	N