ECS 315: Probability and Random Processes	2018/1

HW 10 — Due: Nov 22, 4 PM

Lecturer: Prapun Suksompong, Ph.D.

Instructions

(a) This assignment has 5 pages.

(b) (1 pt) Work and write your answers **directly on these sheets** (not on other blank sheets of paper). Hard-copies are distributed in class.

(c) (1 pt) Write your first name and the last three digits of your student ID on the upper-right corner of this page.

(d) (8 pt) Try to solve all problems.

(e) Carefully write down all the steps that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.

Problem 1 (Yates and Goodman, 2005, Q3.2.1)**.** The random variable X has probability density function

$$f_X(x) = \begin{cases} cx & 0 \le x \le 2, \\ 0, & \text{otherwise.} \end{cases}$$

Use the pdf to find the following quantities.

(a) the constant c

(b) $P[0 \le X \le 1]$

(c) $P[-1/2 \le X \le 1/2]$.

(d) the cdf $F_X(x)$.

Problem 2 (Modified from Yates and Goodman, 2005, Q3.1.3). The CDF of a random variable W is

$$F_W(w) = \begin{cases} 0, & w < -5, \\ (w+5)/8, & -5 \le w < -3, \\ 1/4, & -3 \le w < 3, \\ 1/4 + 3(w-3)/8, & 3 \le w < 5, \\ 1, & w \ge 5. \end{cases}$$

(a) Is W a continuous random variable?

(b) What is $P[W \le 4]$?

(c) What is $P[-2 < W \le 2]$?

(d) What is $P[W > 0]$?

(e) What is the value of a such that $P[W \le a] = 1/2$?

Problem 3 (Yates and Goodman, 2005, Q3.2.3). The CDF of random variable W is

$$
F_W(w) = \begin{cases}
0, & w < -5, \\
(w + 5)/8, & -5 \le w < -3, \\
1/4, & -3 \le w < 3, \\
1/4 + 3(w - 3)/8, & 3 \le w < 5, \\
1, & w \ge 5.
\end{cases}
$$

Find its pdf $f_W(w)$.

Problem 4 (Yates and Goodman, 2005, Q3.3.4). The pdf of random variable Y is

$$
f_Y(y) = \begin{cases}
y/2 & 0 \le y < 2, \\
0, & \text{otherwise.}
\end{cases}
$$

(a) Find $\mathbb{E}[Y]$.

(b) Find $\operatorname{Var} Y$.

Problem 5 (Yates and Goodman, 2005, Q3.3.6). The cdf of random variable V is

$$F_V(v) = \begin{cases} 0 & v < -5, \\ (v+5)^2/144, & -5 \le v < 7, \\ 1 & v \ge 7. \end{cases}$$

(a) What is $f_V(v)$?

(b) What is $\mathbb{E}[V]$?

(c) What is $\mathrm{Var}[V]$?

(d) What is $\mathbb{E}[V^3]$?