ECS 315: In-Class Exercise # 9

Date: **25** / **09** / 2018	
Name	ID (last 3 digits)
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Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1) [Digital Communication] A certain binary-symmetric channel has a crossover probability (bit-error rate) of 0.4. Assume bit errors occur independently. Your answers for parts (a) and (b) should be of the form X.XXXX.

a) Suppose we input bit sequence "1010101" into this channel.

i) What is the probability that the output is "1000001"?

$$(1-p)\times(1-p)\times p\times(1-p)\times p\times(1-p)\times(1-p) = (1-p)^5p^2 = 0.6^5\times0.4^2 = \frac{972}{78125} \approx 0.0124$$

ii) What is the probability that exactly 4 bits are in error at the channel output?

$$\binom{7}{4}p^4(1-p)^3 = \frac{7\times\cancel{6}\times5}{3\times\cancel{2}\times1}0.4^4\,0.6^3 = 0.1935$$

iii) What is the probability that there is at least one bit error at the channel output?

$$1-(1-p)^7 = 1-0.6^7 \approx 0.9720$$

b) Suppose we keep inputting bits into this channel. What is the probability that the **first** bit error at the output occurs on the fourth bit?

$$(1-p)^3p = 0.6^3\times0.4 = 0.0864$$

c) (Optional) Suppose the input bits are generated by flipping a fair coin 7 times. Heads and tails are represented by 1 and 0, respectively.

Let A be the event that the output of the channel is "1000001".

Let B_1 be the event that the input of the channel is "1100011". Same reasoning as 1.a.i gives $\Rightarrow P(A|B_1) = p^2(1-p)^5$

Let B_2 be the event that the input of the channel is "1011101". $\Rightarrow P(A|B_2) = p^3(1-p)^4$ $\Big\}$ (*)

Compare $P(B_1 | A)$ and $P(B_2 | A)$. (Which one is larger? Explain.)

$$P(B_1|A) = \frac{P(B_1\cap A)}{P(A)} = \frac{P(A|B_1)P(B_1)}{P(A)}$$

$$P(B_2|A) = \frac{P(B_2\cap A)}{P(A)} = \frac{P(A|B_2)P(B_2)}{P(A)}$$

Note that the coin is fair; therefore, $P(B_1) = P(B_2) = \frac{1}{2^7}$.

Also, the division by $P(A)$ happens in both $P(B_1|A)$ and $P(B_2|A)$.

Hence, $\frac{P(B_1|A)}{P(B_2|A)} = \frac{P(A|B_1)}{P(A|B_2)}$. (**)

$$\frac{P(B_1|A)}{P(B_2|A)} \overset{(**)}{=} \frac{P(A|B_1)}{P(A|B_2)} \overset{(*)}{=} \frac{p^2(1-p)^5}{p^3(1-p)^4} = \frac{1-p}{p} = \frac{6}{4} > 1$$

$$\Rightarrow P(B_1|A) > P(B_2|A)$$