ECS 315: In-Class Exercise # 20

Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

Date: **29** / **11** / 2018			
Name	ID (last 3 digits)		
Prapun	**5**	**5**	**5**

[F2013] Random variables X and Y have the following joint pmf

$$p_{X,Y}(x,y)=\begin{cases} c(x+y), & x\in\{1,3\} \text{ and } y\in\{1,3\},\\ 0, & \text{otherwise.}\end{cases}$$

a) Find c

$x+y$	$y=1$	$y=3$
$x=1$	2	4
$x=3$	4	6

For a joint pmf, "$\sum=1$"

$\Rightarrow 2c+4c+4c+6c=1$

$16c=1$

$c=1/16$

b) Find the joint pmf matrix $\mathbf{P}_{X,Y}$

$$P_{X,Y}=\begin{array}{c|cc} x\backslash y & 1 & 3\\ 1 & 2/16 & 4/16\\ 3 & 4/16 & 6/16\end{array}=\begin{array}{c|cc} x\backslash y & 1 & 3\\ 1 & 1/8 & 1/4\\ 3 & 1/4 & 3/8\end{array}$$

c) Find $P[X-Y>1]$

$x-y$	$y=1$	$y=3$
$x=1$	0	−2
$x=3$	2	0

only this position satisfies the condition ">1".

$P[X-Y>1]=\frac{1}{4}$

d) Find the pmf $p_X(x)$ and the pmf $p_Y(y)$.

$$P_{X,Y}=\begin{array}{c|cc} x\backslash y & 1 & 3\\ 1 & 2/16 & 4/16\\ 3 & 4/16 & 6/16\\ \hline \sum & 6/16 & 10/16\\ & 3/8 & 5/8\end{array}$$

$$p_X(x)=\begin{cases} 3/8, & x=1,\\ 5/8, & x=3,\\ 0, & \text{otherwise.}\end{cases}$$

$$p_Y(y)=\begin{cases} 3/8, & y=1,\\ 5/8, & y=3,\\ 0, & \text{otherwise.}\end{cases}$$

Note that X and Y are identically distributed.

e) Find $\mathrm{Cov}[X,Y]$.

$$=\mathbb{E}[XY]-\mathbb{E}X\,\mathbb{E}Y=5-\left(\frac{9}{4}\right)^2=\frac{80-81}{16}=-\frac{1}{16}$$

$$\mathbb{E}X=\sum_x x\,p_X(x)=1\times\frac{3}{8}+3\times\frac{5}{8}=\frac{3+15}{8}=\frac{18}{8}=\frac{9}{4}=\mathbb{E}Y$$

$$xy\,p_{X,Y}(x,y)\quad\begin{array}{c|cc} x\backslash y & 1 & 3\\ 1 & 1\times\frac{1}{8} & 3\times\frac{2}{8}\\ 3 & 3\times\frac{2}{8} & 9\times\frac{3}{8}\end{array}\Rightarrow\mathbb{E}[XY]=\sum_{(x,y)}xy\,p_{X,Y}(x,y)=\frac{1+6+6+27}{8}=\frac{40}{8}=5$$