ECS 315: In-Class Exercise # 14

Date: 30 / 10 / 2018			
Name	ID (last 3 digits)		
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Instructions

1. Separate into groups of no more than three persons. **The group cannot be the same as any of your former groups after the midterm.**
2. **Write down all the steps** that you have done to obtain your answers. You may not get full credit even when your answer is correct without showing how you get your answer.
3. **Do not panic.**

1. Suppose $X \sim \mathcal{G}_0\left(\frac{1}{2}\right)$. Plot its cdf $F_X(x)$ on the interval $[-3,3]$.

$S_X = \{0, 1, 2, \dots\}$

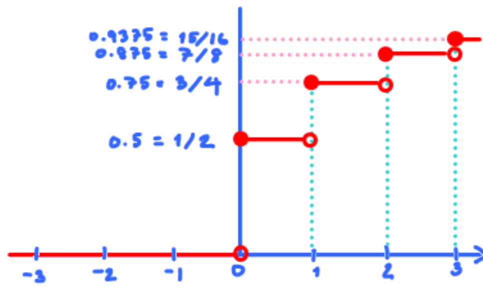
$$p_X(x) = \begin{cases} p(1-p)^x, & x = 0, 1, 2, \dots, \\ 0, & \text{otherwise.} \end{cases}$$

$P[X=0] = \frac{1}{2}$

$P[X=1] = \frac{1}{4}$

$P[X=2] = 1/8$

$P[X=3] = 1/16$



}After this, the graph still increases in even smaller steps. However, we stop here because we are asked to focus on on the interval [-3,3].

2. Arrivals of customers at a local supermarket are modeled by a Poisson process with a rate of $\lambda = 0.5$ customers per minute. Let M be the number of customers arriving between 10:54 AM and 11:00 AM. What is the probability that $M \ge 2$?

$\left.\begin{matrix}\lambda = 0.5 \\ \tau = 6\end{matrix}\right\} \alpha = \lambda\tau = 0.5 \times 6 = 3$

$$p_M(m) = e^{-\alpha}\frac{\alpha^m}{m!}$$

$$P[M<2] = \underbrace{P[M=0]}_{0.0498} + \underbrace{P[M=1]}_{0.1494}$$

$$= e^{-\alpha} + \alpha e^{-\alpha} = (\alpha+1)e^{-\alpha} \approx 0.1991$$

$$P[M \ge 2] = 1 - (\alpha+1)e^{-\alpha} = 1 - 4e^{-3} \approx 0.8009$$

3. Consider (a sequence of independent) Bernoulli trials whose success probability for each trial is $1/5$. For each of the random variables defined below, indicate the **name and the parameter(s)** of the family it belongs to.

Random Variable	Family
$K =$ the number of failures until the first success occurs. (may fail 0 time.)	$\mathcal{G}_0(1/5)$
$N =$ the number of successes among the first 7 trials.	$\mathcal{B}(7, 1/5)$
(Optional) Suppose we know that [there is exactly one success during the first 7 trials] ← A. Let $M =$ the trial position in which that success occurs	$\mathcal{U}(\{1,2,\dots,7\})$

This suggests conditional probability

Let B_k be the event that the results of the Bernouli trials are F..FSF..F (S = success, F = failure, S at the k^{th} position)

$$P[M=k|A] = P(B_k|A) = \frac{P(B_k \cap A)}{P(A)} = \frac{p^1\cancel{(1-p)^6}}{\binom{7}{1}p^1\cancel{(1-p)^6}}$$